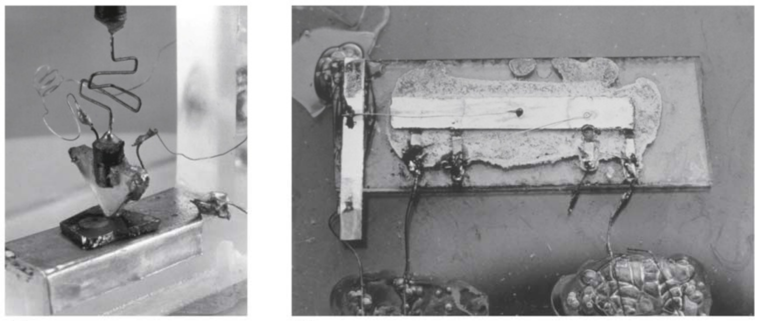
Chapter 4 Physics of Bipolar Transistors

- 4.1 General Considerations
- 4.2 Structure of Bipolar Transistor
- 4.3 Operation of Bipolar Transistor in Active Mode
- 4.4 Bipolar Transistor Models
- 4.5 Operation of Bipolar Transistor in Saturation Mode
- 4.6 The PNP Transistor

Bipolar Transistor 雙極性電晶體

- Bipolar transistor, invented in 1945 by Shockley, Brattain, and Bardeen at Bell Laboratories, subsequently replaces vacuum tubes in electronic systems and paves the way for integrated circuits (IC). Received the Nobel Prize in Physics, 1956.
- In the chapter, we will study the physics of bipolar transistor and derive ***large*** and ***small signal model***s.
- We aim to understand the physics of the transistor, I/V characteristics, and equivalent model used in circuit analysis and design.

Voltage-Controlled Device as Amplifying Element ⇨ Structure of Bipolar Transistor ⇨ Operation of Bipolar Transistor ⇨ Large-Signal Model ⇨ Small-Signal Model



(a) (b)

FIGURE 1.2 (a) First transistor (Property of AT&T Archives. Reprinted with permission of AT&T.) and (b) first integrated circuit (Courtesy of Texas Instruments.)

FIGURE 1.71 Engineer holding processed 12-inch wafer (Photograph courtesy of the Intel Corporation.)

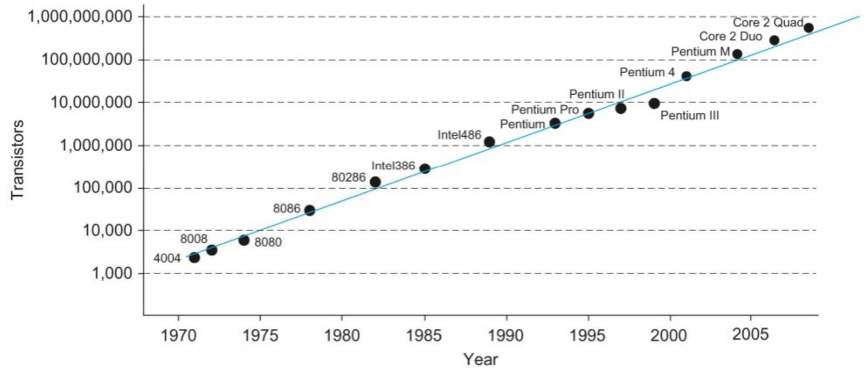
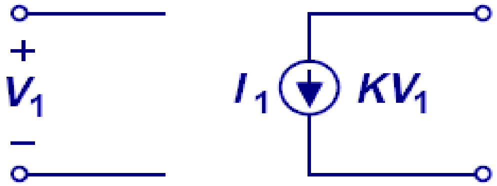


FIGURE 1.4 Transistors in Intel microprocessors [Intel10]

Voltage-Dependent Current Source

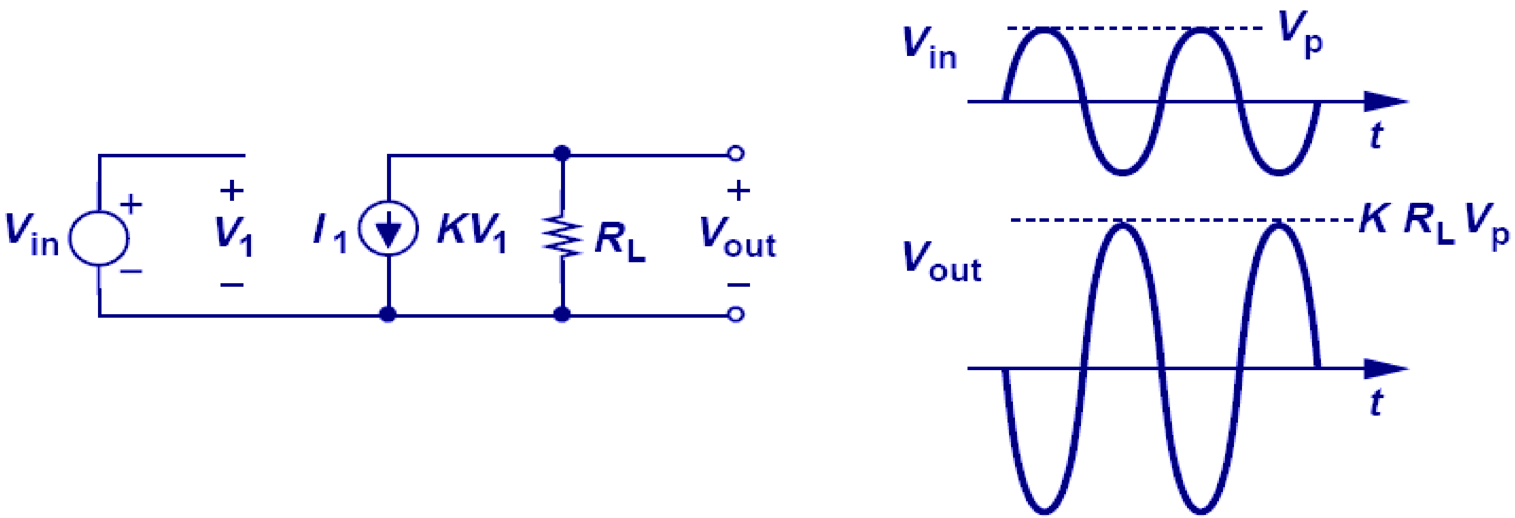
- Bipolar transistor can be viewed as a ***voltage-dependent current source***.
- A voltage-dependent current source can act as an amplifier.
- If $KR_L > 1$, then the signal is amplified.

$$A_V = \frac{V_{out}}{V_{in}} = -KR_L$$



電壓相依電流源

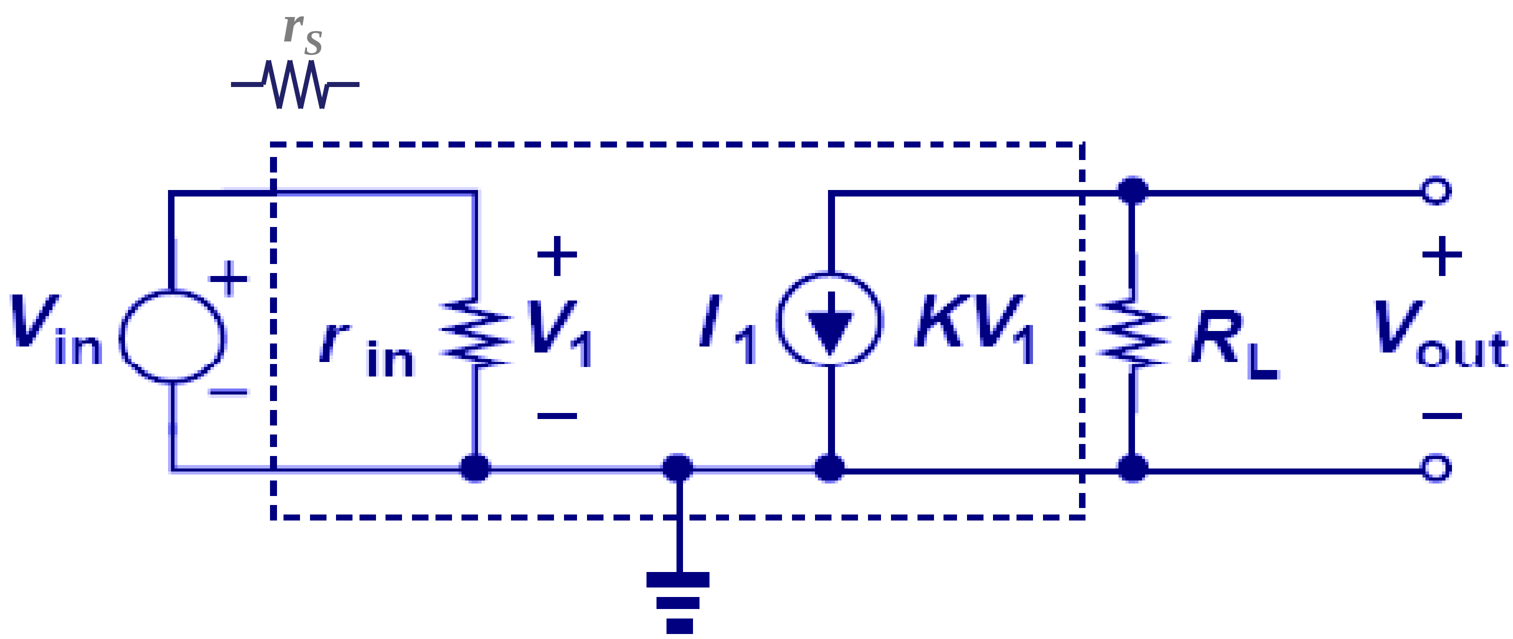
(a)



(b)

Voltage-Dependent Current Source with Input Resistance

- Regardless of the input resistance, r_{in}, the magnitude of amplification remains unchanged. (Example 4.1)

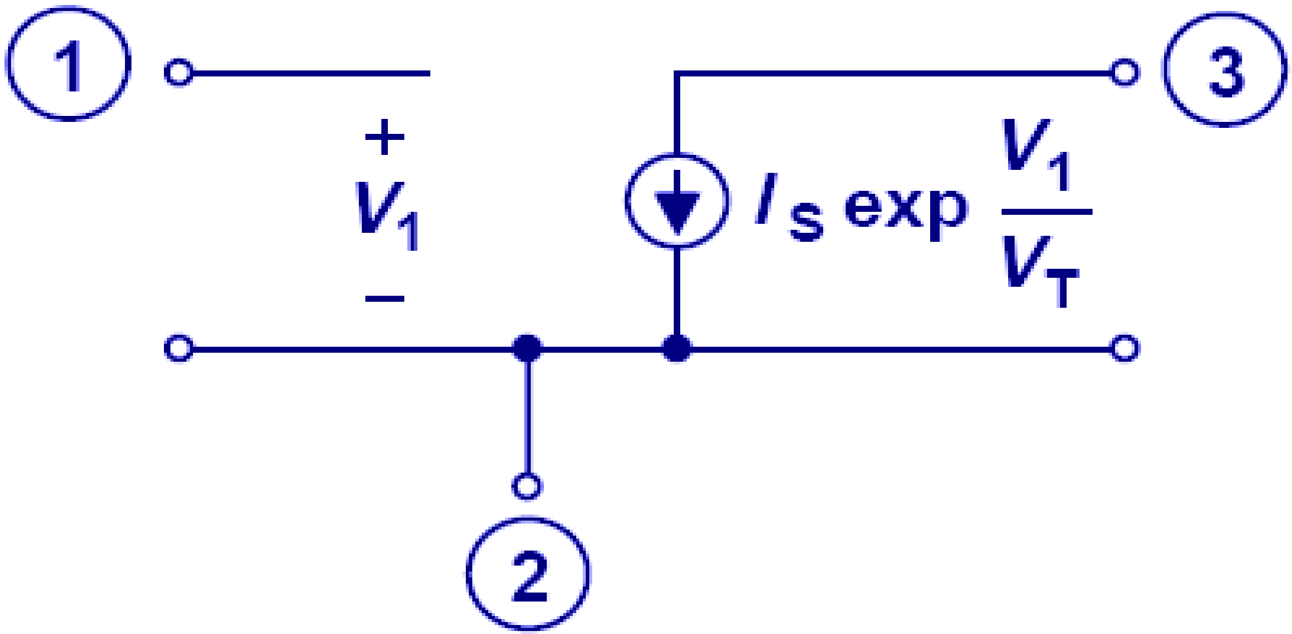


V_{in} usually exists a source resistance, r_S, which will degrade the gain by a factor $\frac{r_{in}}{r_{in}+r_S}$

$$A_V = \frac{V_{out}}{V_{in}} = -KR_L \frac{r_{in}}{r_{in}+r_S}$$

Exponential Voltage-Dependent Current Source

- A three-terminal exponential voltage-dependent current source is shown here.
- Ideally, bipolar transistor can be modeled as such.



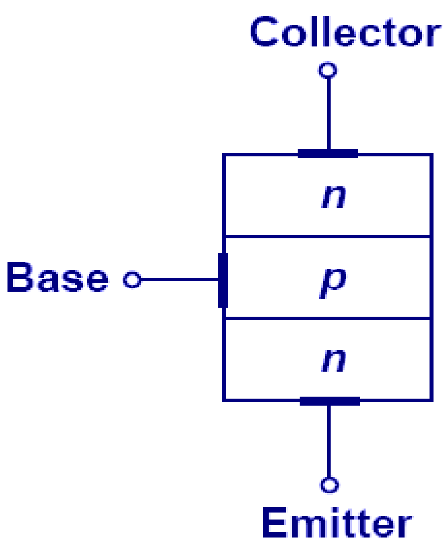
Structure and Symbol of Bipolar Transistor

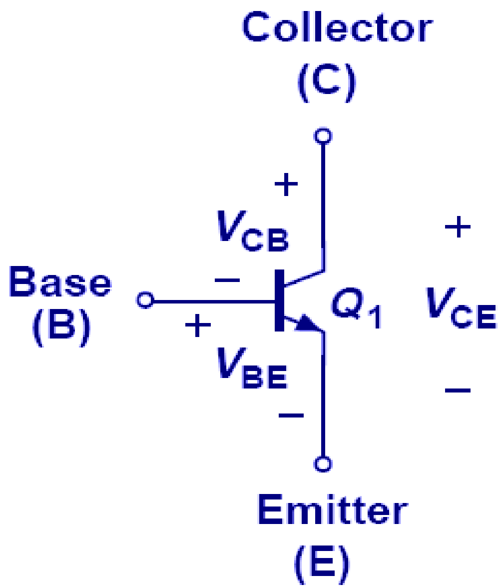
- Bipolar transistor can be thought of as a sandwich of three doped Si regions. The outer two regions are doped with the same polarity, while the middle region is doped with opposite polarity.
- Two possible structures: ***npn*** and ***pnp***.
- The three terminals are: ***base***, ***emitter***, and ***collector***.

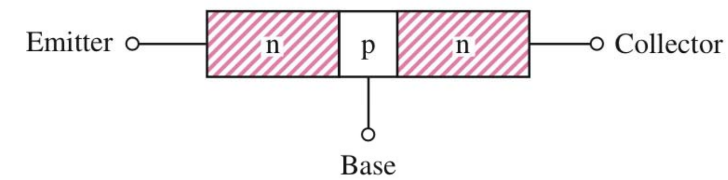
B: 基極

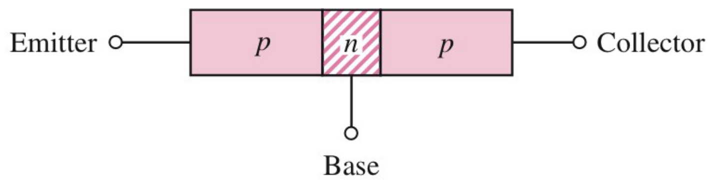
E: 射極

C: 集極









Proper operation requires a thin base region, ~10nm

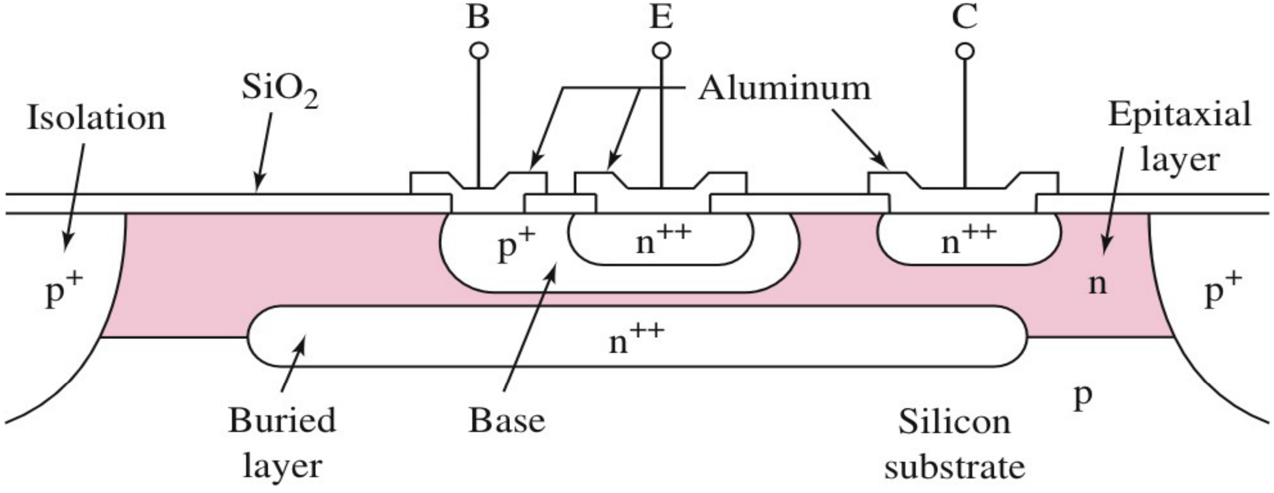
Cross Section of a Conventional IC *npn* BJT

- The device is not symmetrical electrically
- The geometries of the emitter and collector regions are not the same, and the impurity doping concentrations in the three regions are substantially different.
- Impurity concentration:

$$E : 10^{19}\,cm^{-3}$$

$$B : 10^{17}\,cm^{-3}$$

$$C : 10^{15}\,cm^{-3}$$



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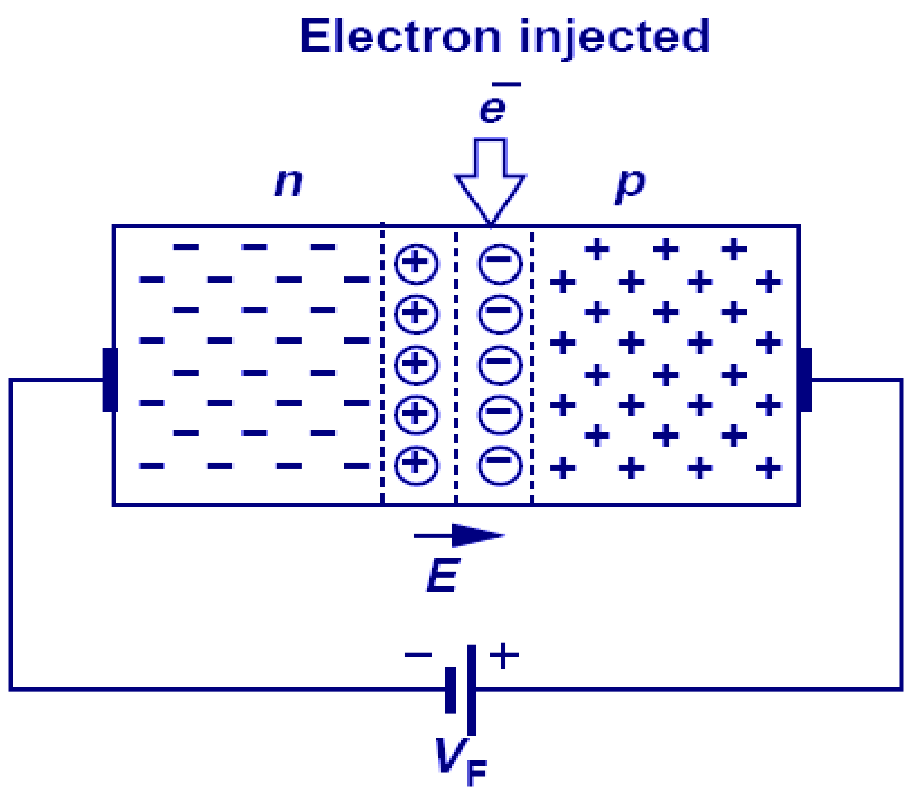
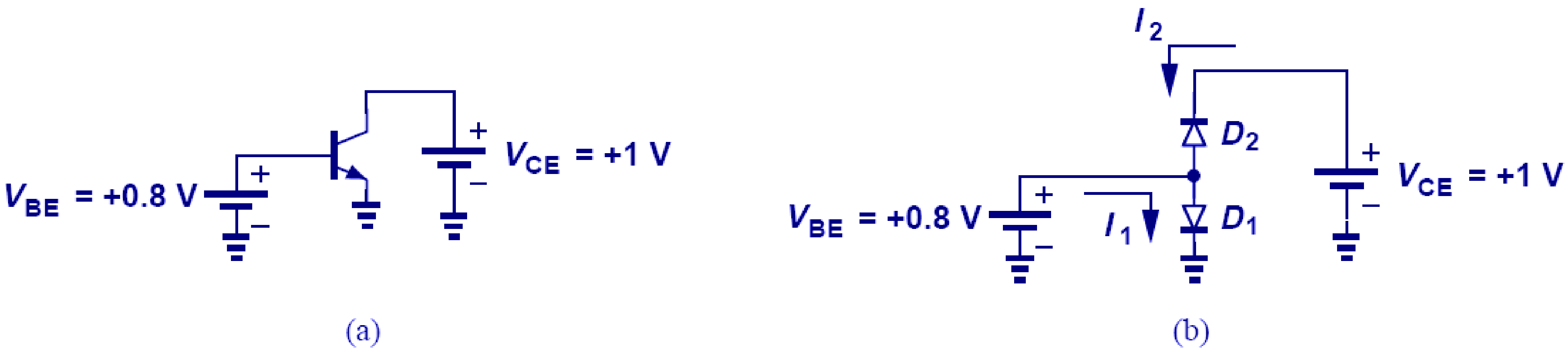
Injection of Carriers

- Reverse biased *PN* junction creates a large electric field that sweeps any injected minority carriers to their majority region.
- This ability proves essential in the proper operation of a bipolar transistor.



Forward Active Region

- ***Forward active region***: $V_{BE} > 0$, $V_{BC} < 0$, *i.e.*, base-emitter junction is forward-biased and base-collector junction reverse-biased
- Transistor acts as a voltage-controlled current source.
  - the current flow from the emitter to the collector can be viewed as a current source tied between these two terminals, I_{CE}
  - this current is controlled by the voltage difference between the base and the emitter, V_{BE}.
- Figure (b) presents a '*wrong*' way of modeling figure (a).

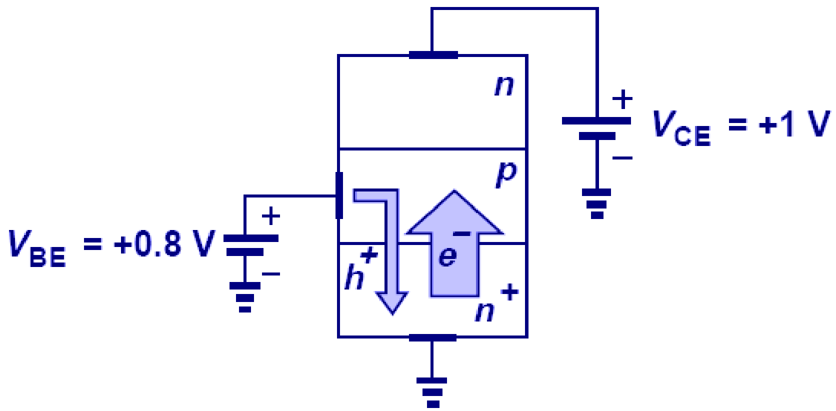


$V_{BE} = 0.8$ V, $V_{BC} = -0.2$ V; D_1 is ON, D_2 is 'OFF'

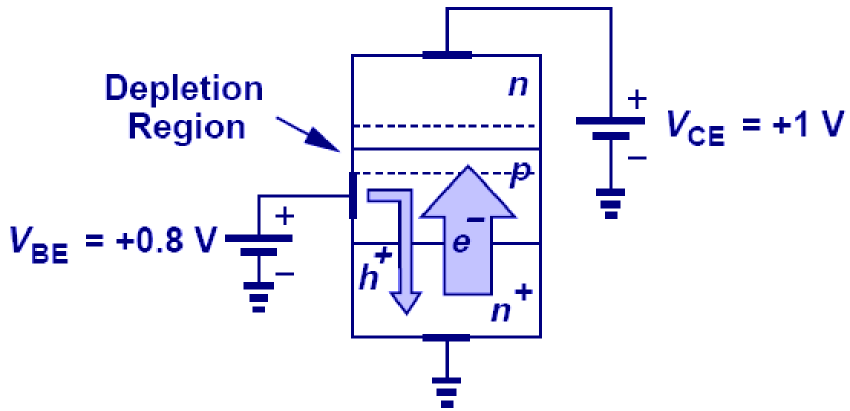
Accurate Bipolar Representation

- Collector also carries current due to carrier injection from base.

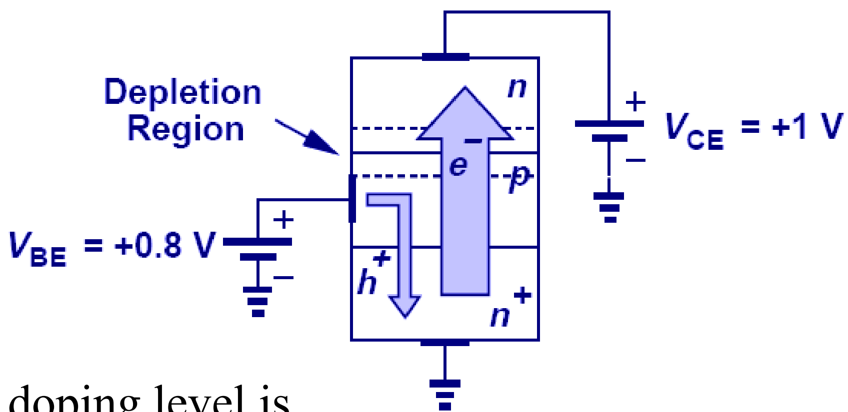
  remember that the *base* must be very thin



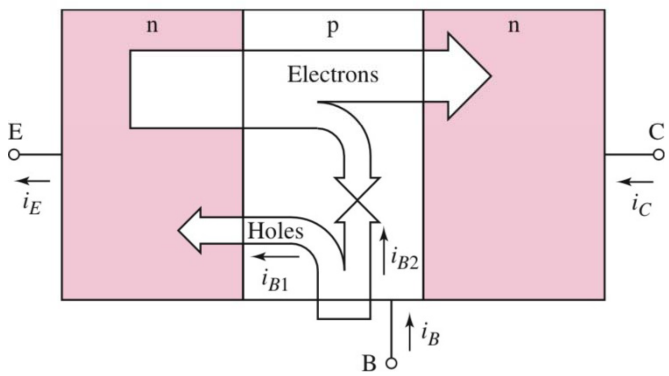
(a)



(b)

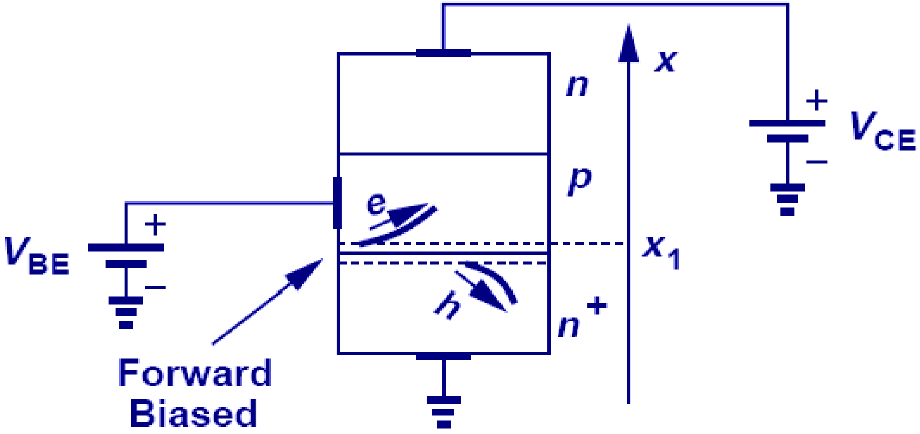


(c)

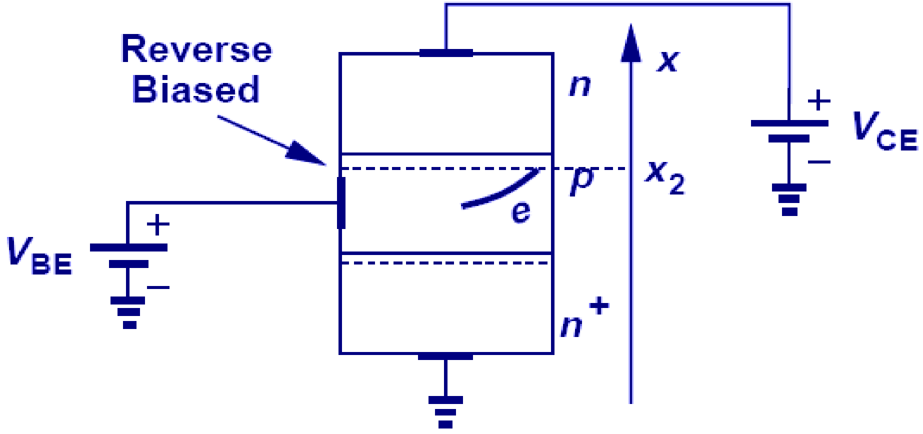


n^+ means the emitter doping level is much greater than base

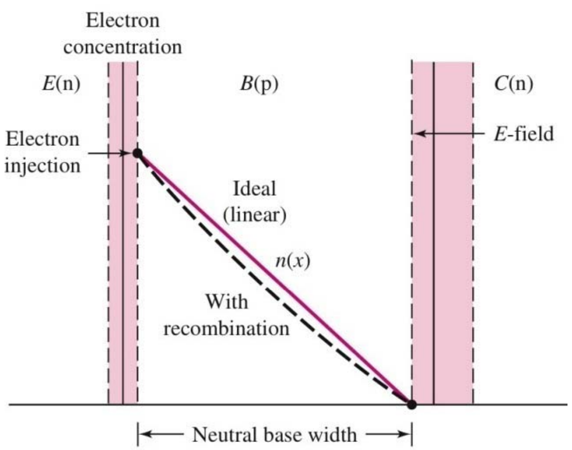
Carrier Transport in Base

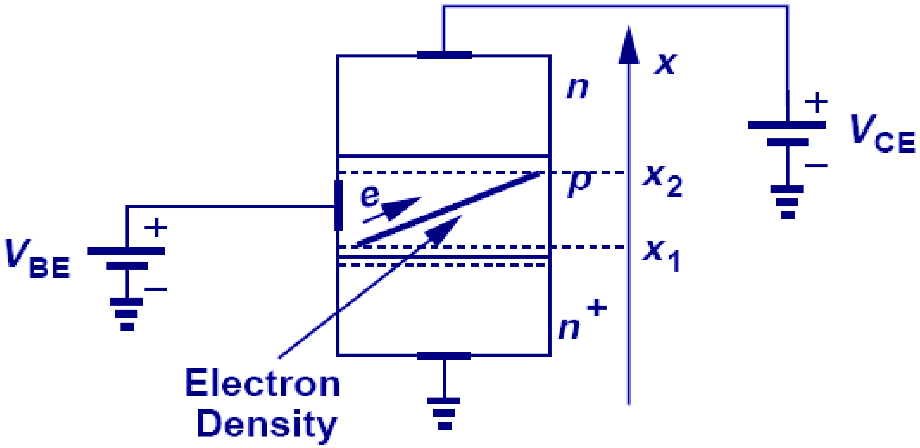


(a)



(b)





(c)

The electron density gradient in the base provides the diffusion of electrons.

Collector Current

- Applying the law of diffusion, we can determine the charge flow across the base region into the collector.
- The equation shows that the transistor is indeed a voltage-controlled element, thus a good candidate as an amplifier.

V_{BE} is the input voltage, I_C is the output current

$$I_C = \frac{A_E q D_n n_i^2}{N_B W_B}\left(\exp\frac{V_{BE}}{V_T} - 1\right)$$

$$I_C = I_S \exp\frac{V_{BE}}{V_T}, \quad \text{assuming} \quad \exp\frac{V_{BE}}{V_T} >> 1$$

$$I_S = \frac{A_E q D_n n_i^2}{N_B W_B}$$

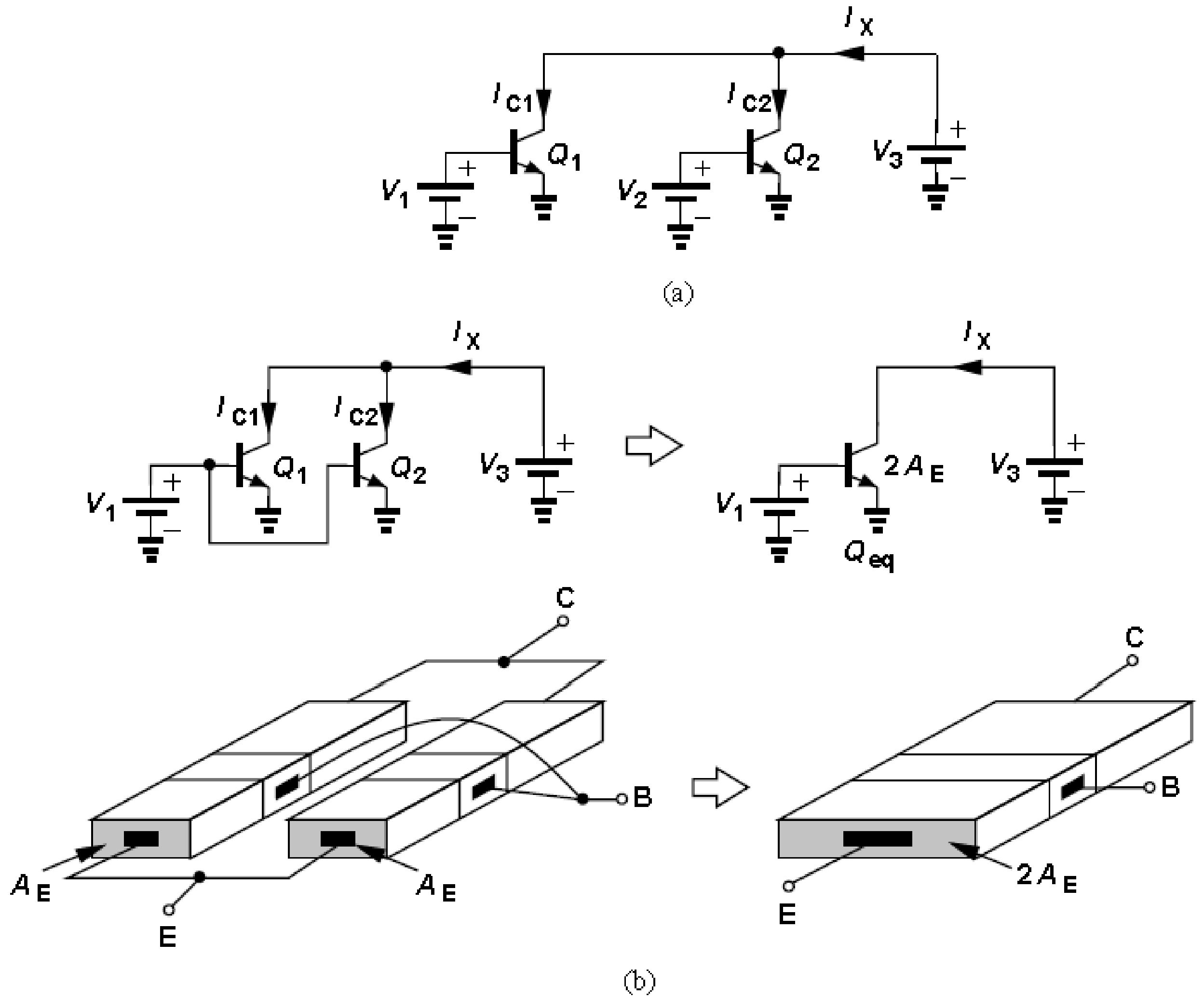
check these equations in chapter 2
remember this one is enough now

$$I_C = I_s(\exp\frac{V_{BE}}{V_T} - 1)$$

Parallel Combination of Transistors

- When two transistors are put in parallel and experience the same potential across all three terminals, they can be thought of as a single transistor with twice the emitter area. *(very common seen in IC layout)*

Fig. 4.9

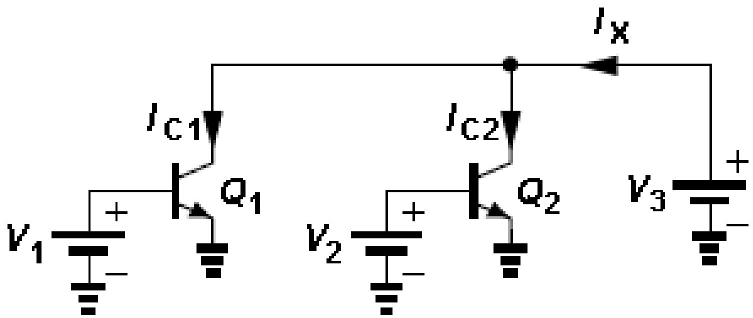


Example 4.2

Determine the current I_X in Fig. 4.9(a) if Q_1 and Q_2 are identical and operate in the active mode and $V_1 = V_2$.

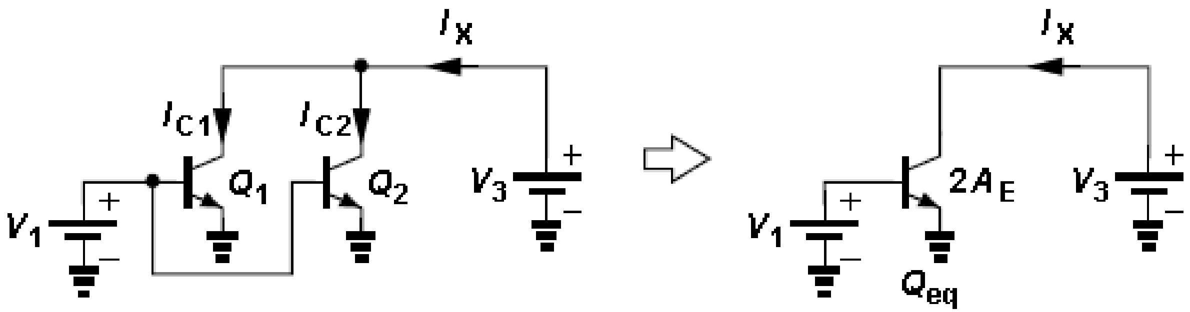
Solution

Since $I_X = I_{C1} + I_{C2}$, we have



$$I_X \approx 2\frac{A_E q D_n n_i^2}{N_B W_B}\exp\frac{V_1}{V_T}.$$

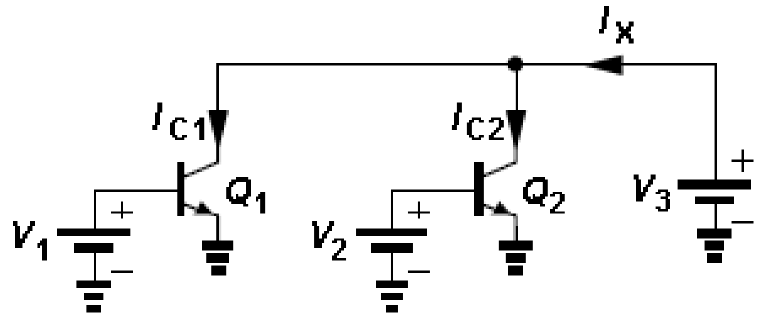
This result can also be viewed as the collector current of a single transistor having an emitter area of $2A_E$. In fact, redrawing the circuit as shown in Fig. 4.9(b) and noting that Q_1 and Q_2 experience identical voltages at their respective terminals, we say the two transistors are "in parallel," operating as a single transistor with twice the emitter area of each.



Example 4.3

In the circuit of Fig. 4.9(a), Q_1 and Q_2 are identical and operate in the active mode. Determine V_1 - V_2 such that $I_{C1} = 10\ I_{C2}$.

Solution



From Eq. (4.9), we have

$$\frac{I_{C1}}{I_{C2}} = \frac{I_S \exp \dfrac{V_1}{V_T}}{I_S \exp \dfrac{V_2}{V_T}},$$

and hence

$$\exp \frac{V_1 - V_2}{V_T} = 10.$$

That is,

$$V_1 - V_2 = V_T \ln 10 \approx 60\text{mV} \quad \text{at} \quad T = 300^\circ\text{K}$$

$(\ln 10 = 2.30)$

Identical to Eq. (2.109), this result is, of course, expected because the exponential dependence of I_C upon V_{CE} indicates a behavior similar to that of diodes. We therefore consider the base-emitter voltage of the transistor relatively constant and approximately equal to 0.8 V for typical collector current levels.

Example 4.4

Typical discrete bipolar transistors have a large area, e.g., 500 x 500 μm^2, whereas modern integrated devices may have an area as small as 0.5 x 0.2 μm^2. Assuming other device parameters are identical, determine the difference between the base-emitter voltage of two such transistors for equal collector currents.

Solution

From Eq. (4.9), we have $V_{BE} = V_T \ln(I_C/I_S)$ and hence

$$V_{BEint} - V_{BEdis} = V_T \ln \frac{I_{S1}}{I_{S2}},$$

where $V_{BEint} = V_T \ln(I_{C2}/I_{S2})$ and $V_{BEdis} = V_T \ln(I_{C1}/I_{S1})$ denote the base-emitter voltages of the integrated and discrete devices, respectively. Since $I_S \propto A_E$,

$$I_S = \frac{A_E q D_n n_i^2}{N_B W_B}$$

$$V_{BEint} - V_{BEdis} = V_T \ln \frac{A_{E2}}{A_{E1}}.$$

For this example, $A_{E2}/A_{E1} = 2.5 \times 10^6$, yielding $V_{BEint} - V_{BEdis} = 383$ mV.

In practice, however, $V_{BEint} - V_{BEdis}$ falls in the range of 100 to 150 mV because of differences in the base width and other parameters. The key point here is that $V_{BE} = 800$ mV is a reasonable approximation for integrated transistors and should be lowered to about 700 mV for discrete devices.

Example 4.5

Determine the output voltage in Fig. 4.10 if $I_S = 5 \times 10^{-16}$A.

$$I_C = I_s \cdot \exp \frac{V_{BE}}{V_T}$$

Solution

Using Eq. (4.9), we write $I_C = 1.69$ mA. This current flows through R_L, generating a voltage drop of 1 kΩ x 1.69 mA = 1.69V. Since $V_{CE} = 3\text{V} - I_C R_L$, we obtain $V_{out} = 1.31$V.

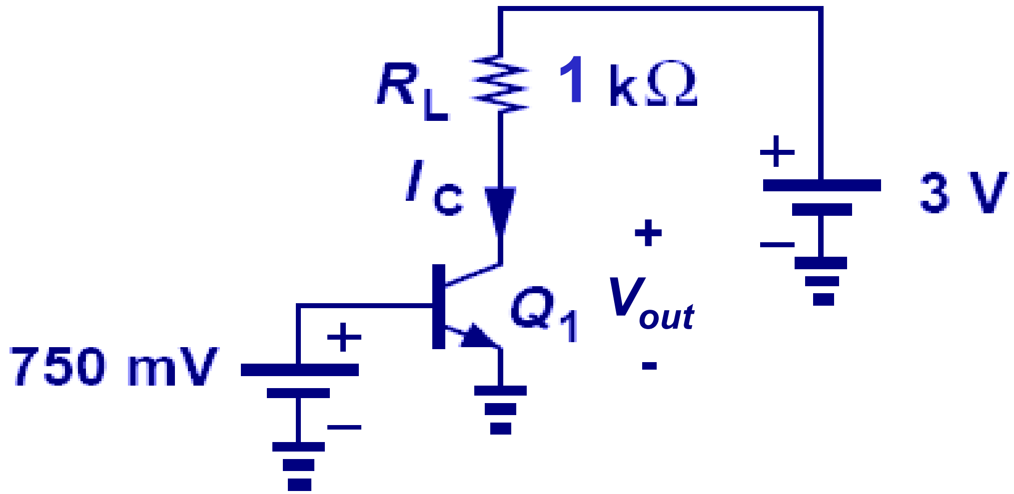
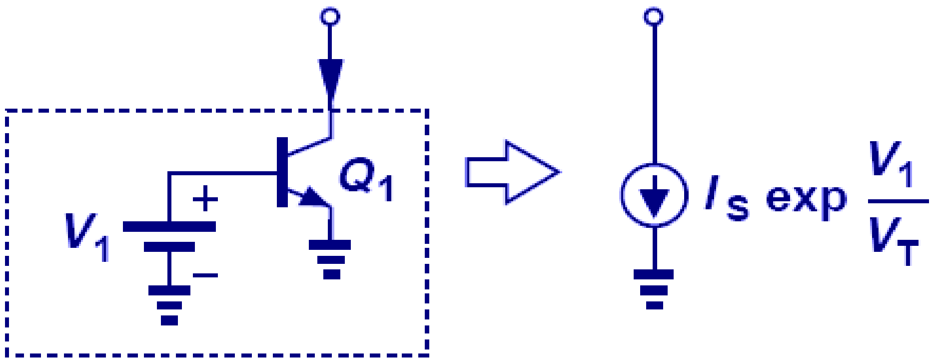


Figure 4.10. Simple stage with biasing.

- Although a transistor is a voltage to current converter, output voltage can be obtained by inserting a load resistor at the output and allowing the controlled current to pass thru it.

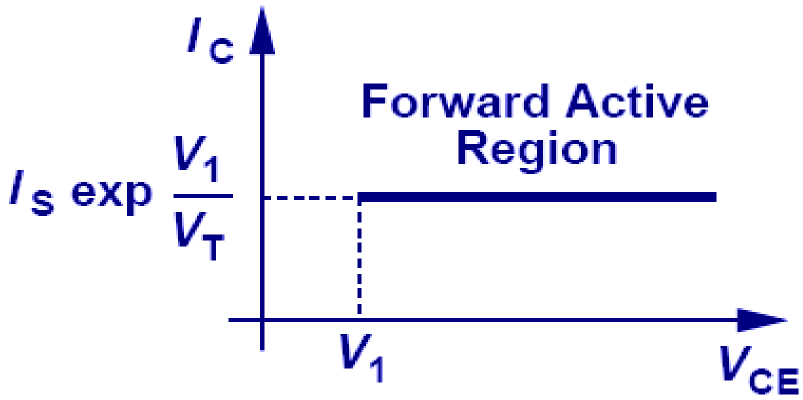
Constant Current Source

- Ideally, the collector current, I_C, does not depend on the collector to emitter voltage, V_{CE}.
- This property allows the transistor to behave as a constant current source when its base-emitter voltage ($V_{BE} = V_1$) is fixed. (for $V_{CE} > V_1$)



(a)

current source



(b)

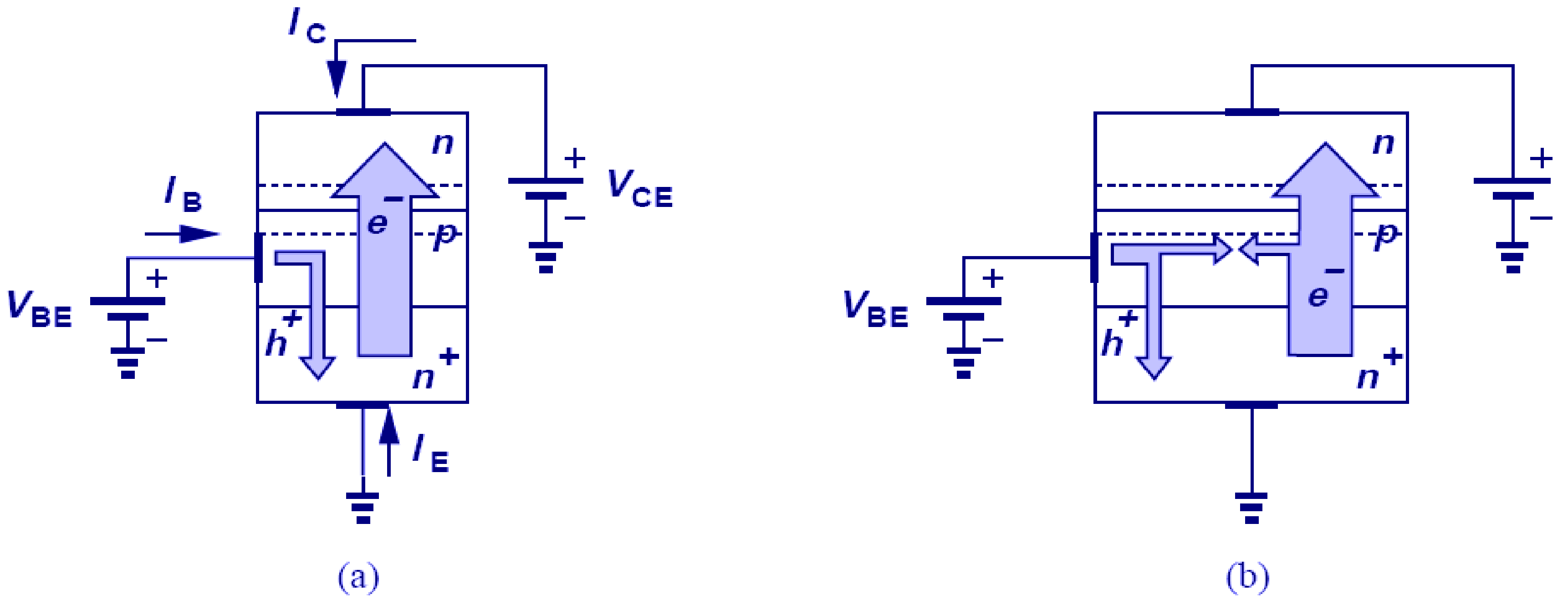
I/V characteristic

Base Current

- Base current consists of two components: (a) Reverse injection of holes into the emitter and (b) recombination of holes with electrons coming from the emitter.

$$I_C = \beta I_B$$

- β is called the "***current gain***" of the transistor because it shows how much the base current is "amplified," typically ranges from 50 to 200.



holes (a) crossing to emitter and (b) recombining with electrons.

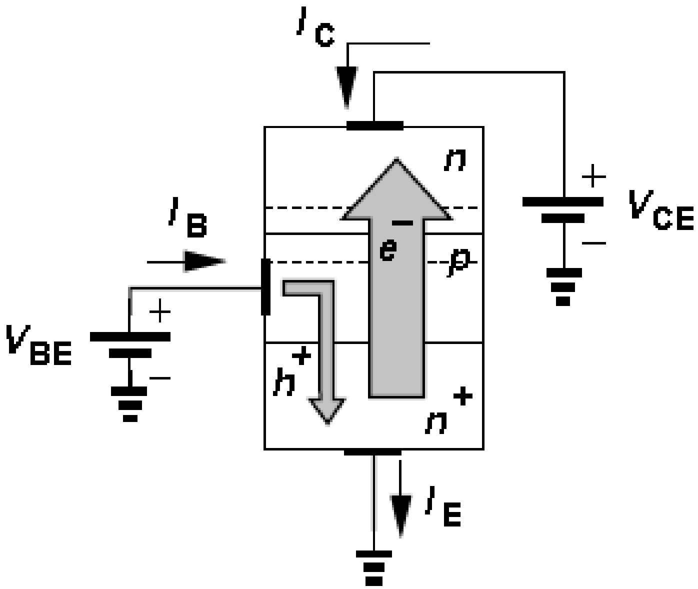
Emitter Current

- Applying Kirchoff's current law to the transistor, we can easily find the emitter current.

$$I_E = I_C + I_B = (\beta + 1) I_B$$

$$I_E = I_C \left(1 + \frac{1}{\beta} \right)$$

$$\beta = \frac{I_C}{I_B}$$



Summary of Currents

$$I_C = I_S \exp \frac{V_{BE}}{V_T}$$

$$I_B = \frac{1}{\beta} I_S \exp \frac{V_{BE}}{V_T}$$

$$I_E = \frac{\beta + 1}{\beta} I_S \exp \frac{V_{BE}}{V_T}$$

$$\frac{\beta}{\beta + 1} = \alpha$$

$$I_C = \beta I_B$$

$$I_E = I_C + I_B = (\beta + 1) I_B$$

- It is sometimes useful to write $I_C = \alpha\, I_E$ and $\alpha = \beta/(\beta+1)$.
- For $\beta = 100$, $\alpha = 0.99$, suggesting that $\alpha \approx 1$ and $I_C \approx I_E$ are reasonable approximations.

Example 4.6

A bipolar transistor having $I_S = 5 \times 10^{-16}$ A is biased in the forward active region with V_{BE} = 750 mV. If the current gain varies from 50 to 200 due to manufacturing variations, calculate the minimum and maximum terminal currents of the device.

Solution

For a given V_{BE}, the collector current remains independent of β:

$$I_C = I_S \exp\frac{V_{BE}}{V_T} = 1.685 \text{ mA}$$

The base current varies from $I_C/200$ to $I_C/50$:

$$8.43\mu\text{A} < I_B < 33.7\mu\text{A}$$

On the other hand, the emitter current experiences only a small variation because $(\beta+1)/\beta$ is near unity for large β:

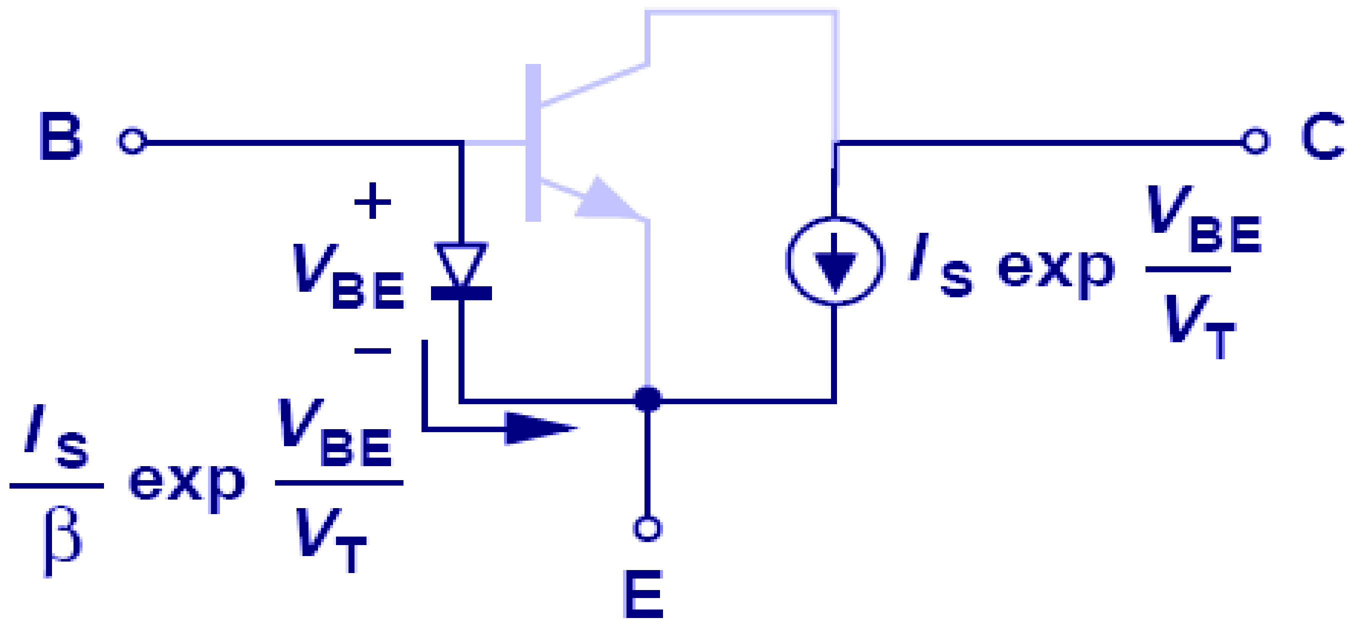
$$1.005 I_C < I_E < 1.02 I_C$$

$$1.693\text{mA} < I_E < 1.719\text{mA}$$

$$1.005 < \alpha < 1.02$$

Bipolar Transistor Large Signal Model

- A diode is placed between base and emitter and a voltage controlled current source is placed between the collector and emitter.



operation in the forward active region

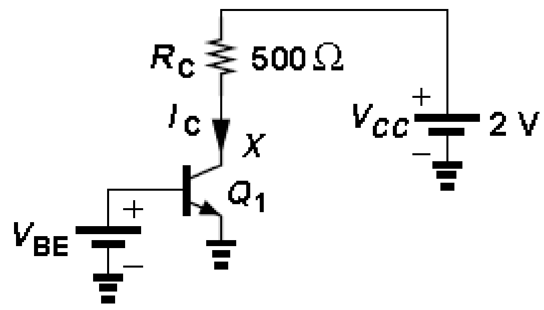
Example 4.7

Consider the circuit shown in Fig. 4.14(a), where $I_{S,Q1} = 5\text{x}10^{-17}$A and $V_{BE} = 800$mV. Assume $\beta = 100$. (a) Determine the transistor terminal currents and voltages and verify that the device indeed operates in the active mode. (b) Determine the maximum value of R_C that permits operation in the active mode.

Solution

(a) Using Eq. (4.23)-(4.25) [slide 21], we have $I_C = 1.153$mA, $I_B = 11.53\mu$A, $I_E =$ 1.165mA. The base and emitter voltages are equal to 800 and 0 mV, respectively. We must now calculate the collector voltage, V_X. Writing a KVL from the 2-V power supply and across R_C and Q_1, we obtain $V_{CC} = I_C R_C + V_X$. That is, $V_X = 1.424$ V. Since the collector voltage is more positive than the base voltage, this junction is reverse-biased and the transistor operates in the active mode.

(b)What happens to the circuit as R_C increases? Since the voltage drop across the resistor, $R_C I_C$, increases while V_{CC} is constant, the voltage at node *X drops*. The device approaches the “edge” of the forward active region if the base-collector voltage falls to zero, i.e., as $V_X \rightarrow 800$mV. Also, $R_C = (V_{CC} - V_X)/I_C$, for $V_X = 800$ mV, yields $R_C = 1041\Omega$. Figure 14(b) plots V_X as a function of R_C.



(a)

Example: Maximum R_C

- As R_C increases, V_X drops and eventually forward biases the collector-base junction. This will force the transistor out of forward active region.
- Therefore, there exists a maximum tolerable collector resistance.

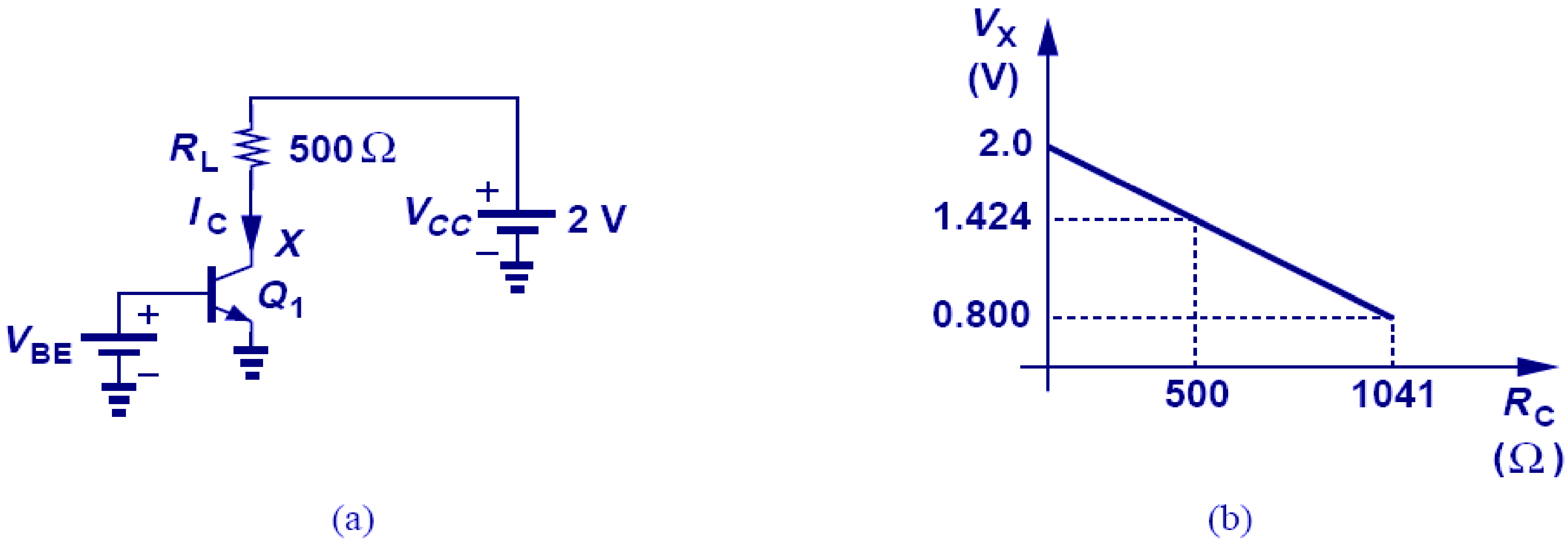
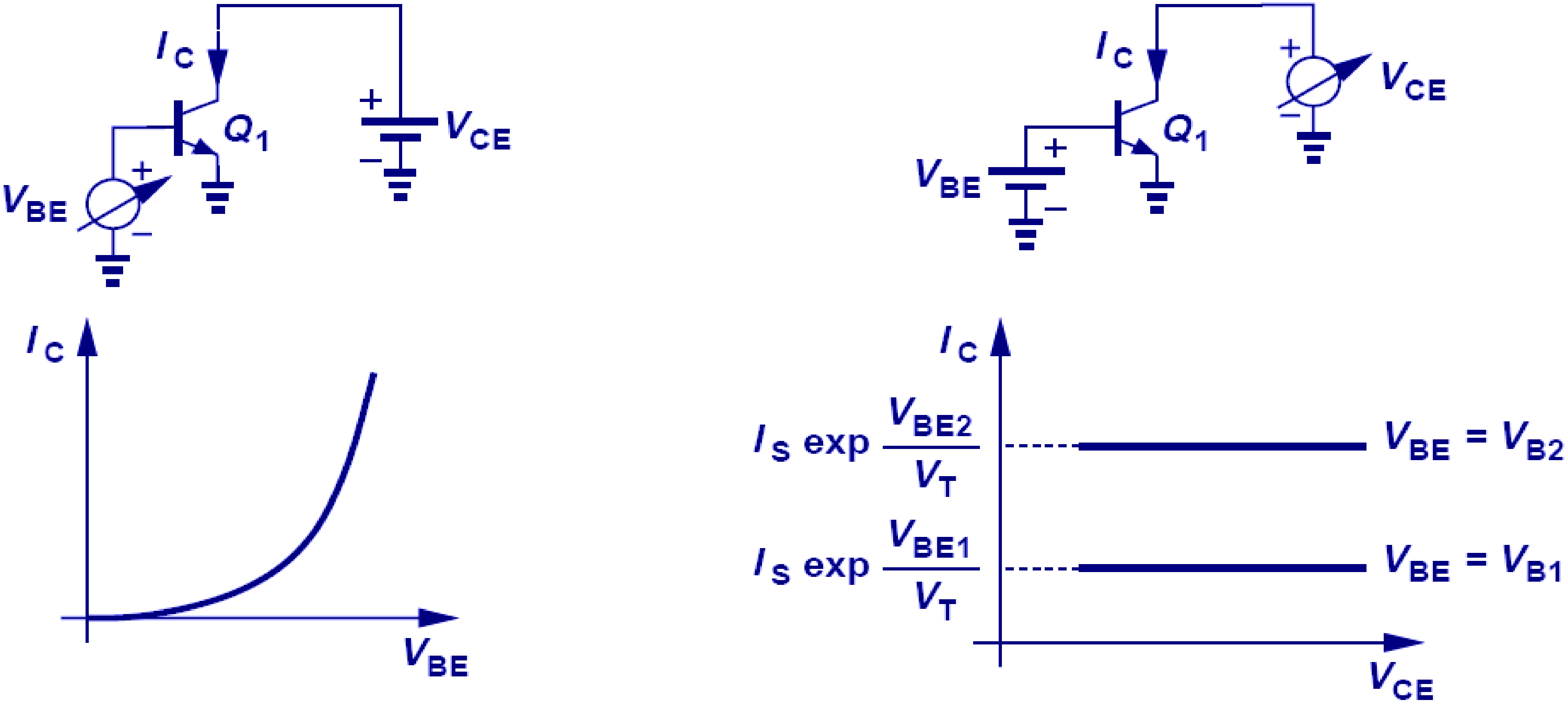


Figure 4.14. (a) Simple stage with biasing, (b) variation of collector voltage as a function of collector resistance.

Characteristics of Bipolar Transistor



(a)

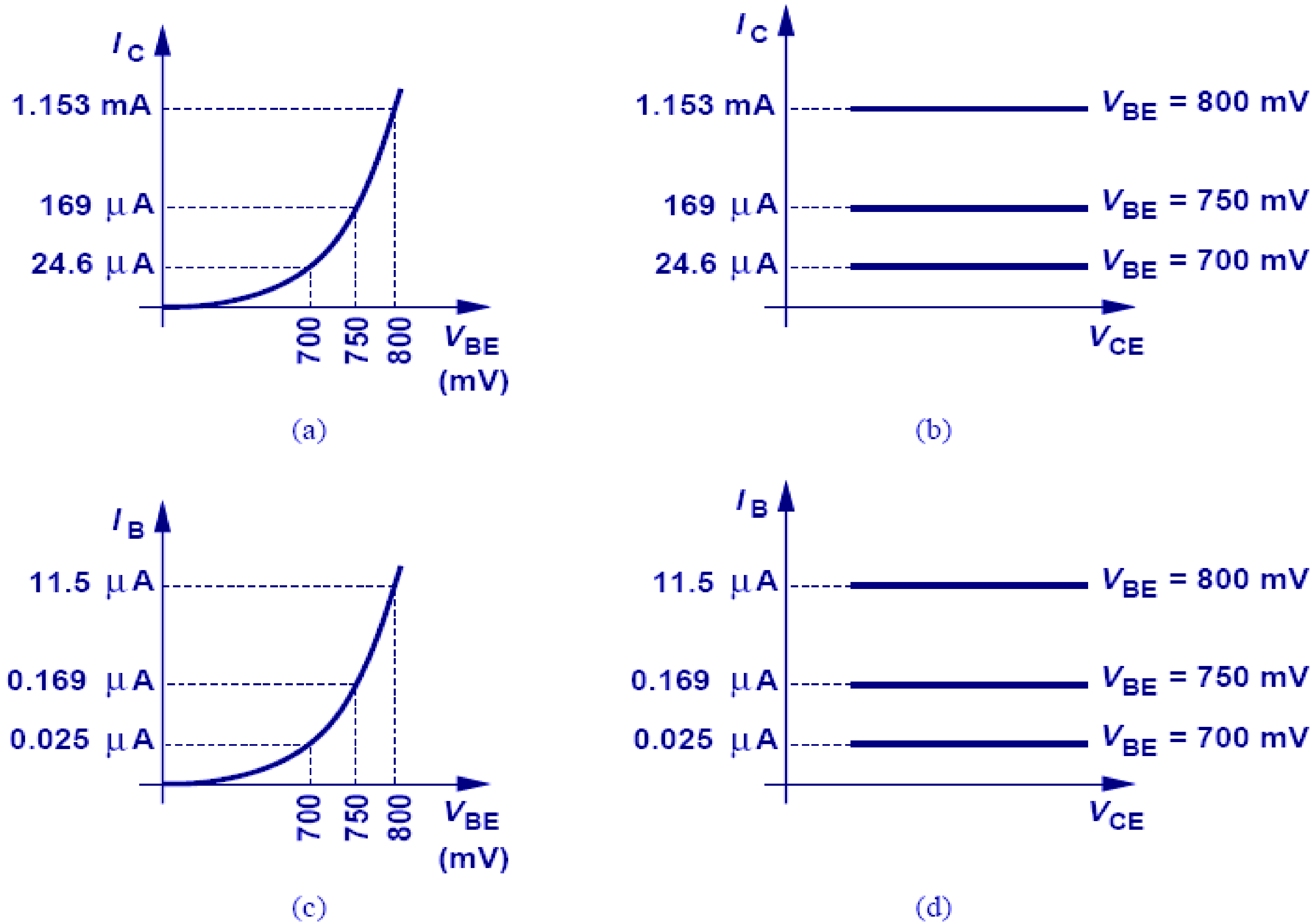
(b)

I_C versus V_{BE}
similar to a diode

I_C versus V_{CE}
resemble a current source

Example 4.8: *I/V* Characteristics

$I_S = 5 \times 10^{-17}$A $\beta = 100$



Transconductance 轉導

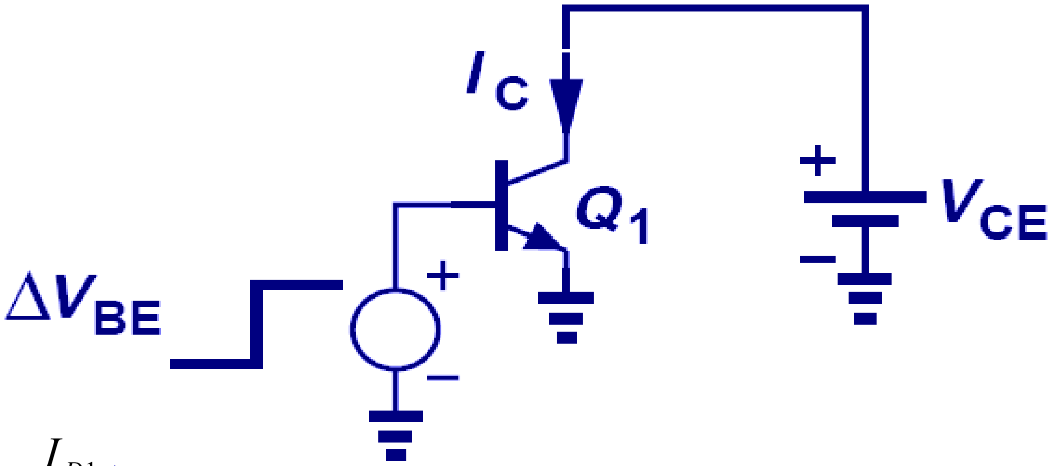
- ***Transconductance***, g_m shows a measure of how well the transistor converts voltage to current.
- It will later be shown that g_m is one of the most important parameters in circuit design.
- The ratio $\Delta I_C/\Delta V_{BE}$ approaches dI_C/dV_{BE} for very small changes and, in the limit, is called the "transconductance," g_m :

$$g_m = \frac{dI_C}{dV_{BE}} = \frac{d}{dV_{BE}}\left(I_S \exp\frac{V_{BE}}{V_T}\right)$$

$$g_m = \frac{1}{V_T} I_S \exp\frac{V_{BE}}{V_T}$$

$$g_m = \frac{I_C}{V_T}$$

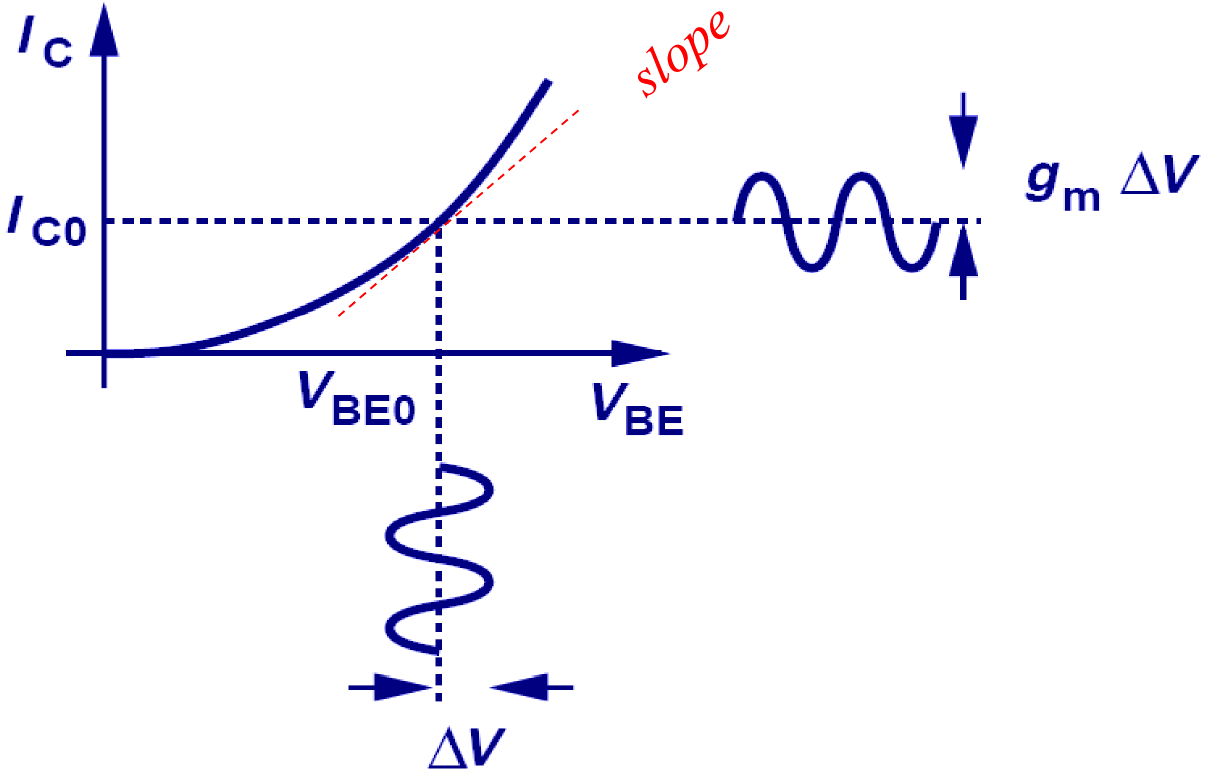
(diode: $\frac{1}{r_d} = \frac{\Delta I_D}{\Delta V_D} = \frac{I_{D1}}{V_T}$)



- As I_C increases, the transistor becomes a better amplifying device by producing larger ΔI_C in response to a given ΔV_{BE}.

Visualization of Transconductance

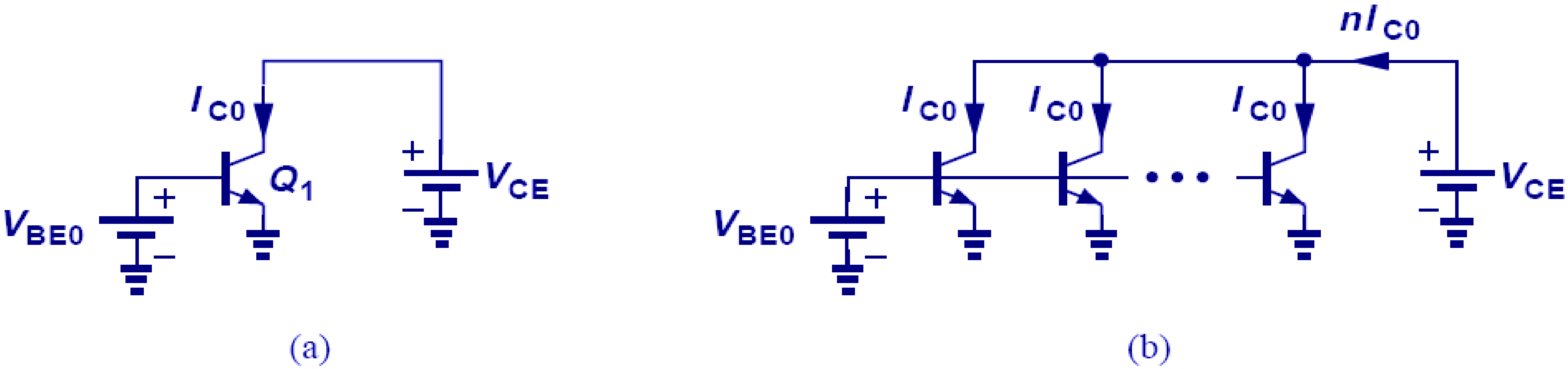
- g_m can be visualized as the slope of I_C versus V_{BE}.
- A large I_C has a large slope and therefore a large g_m.



The ***bias*** (or called ***quiescent***, ***operating***, or just **Q**) point is (V_{BE0}, I_{C0}).
Around the point, a small signal change, ΔV, generate a larger signal, $g_m \Delta V$

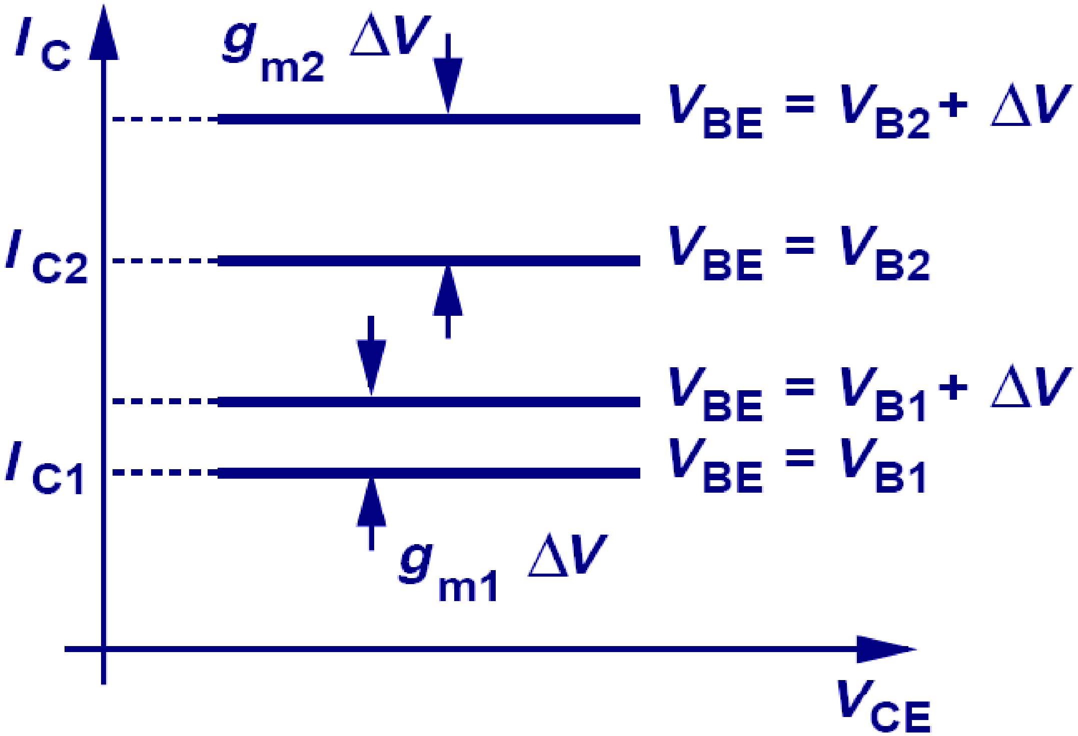
Transconductance and Area

- When the area of a transistor is increased by n, I_S increases by n. For a constant V_{BE}, I_C and hence g_m increases by a factor of n.
- Since $I_S \propto A_E$, I_S is multiplied by the same factor. Thus, $I_C = I_S \exp(V_{BE}/V_T)$ also rises by a factor of n because V_{BE} is constant.



Transconductance and I_c

- The figure shows that for a given V_{BE} swing, the current excursion around I_{C2} is larger than it would be around I_{C1}. This is because g_m is larger for I_{C2}.

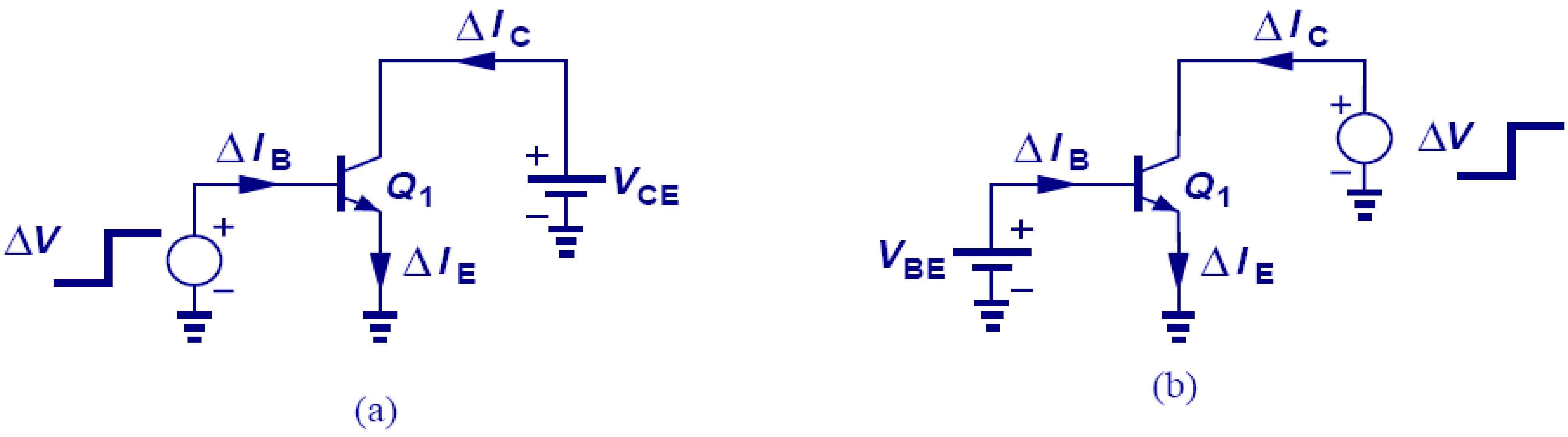


$$g_m = \frac{I_C}{V_T}$$

- g_m is fundamentally a function of I_C rather than I_B.
- For example, if I_C remains constant but β varies, then g_m does not change but I_B does.

Small-Signal Model: Derivation

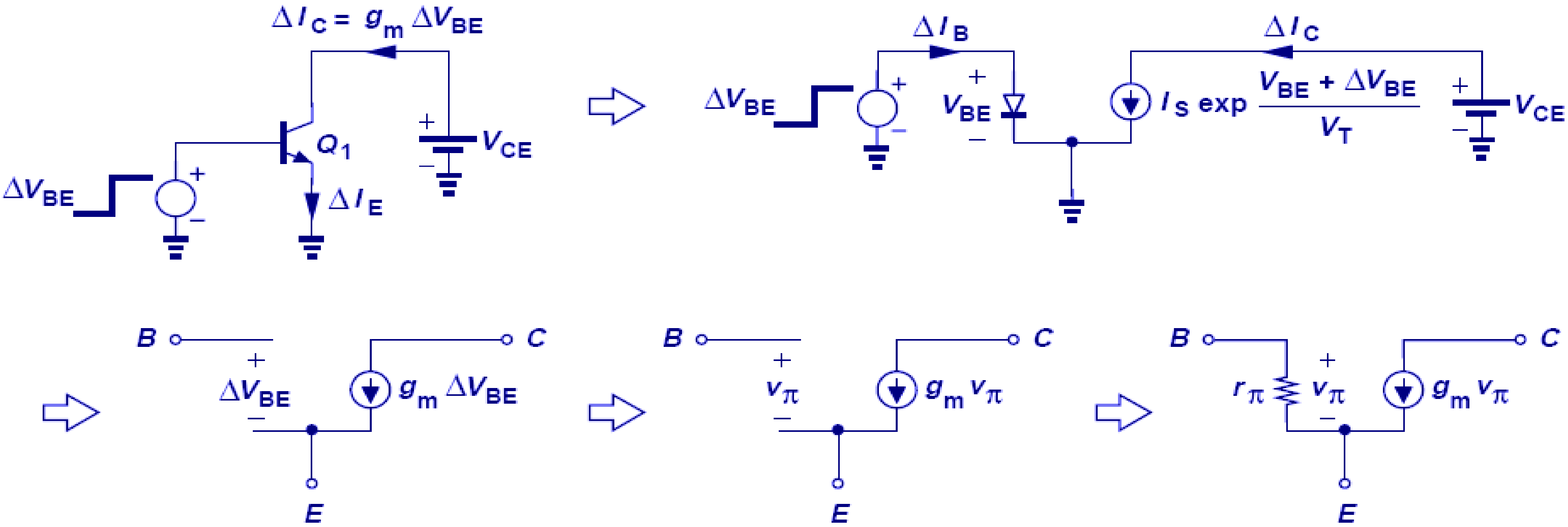
- Nonlinear devices can be reduced to linear devices through the use of the *small-signal model*.
- Small signal model is derived by perturbing voltage difference every two terminals while fixing the third terminal and analyzing the change in current of all three terminals. We then represent these changes with controlled sources or resistors.



Small changes in (a) base-emitter (V_{BE}) and (b) collector-emitter (V_{CE}) voltage.

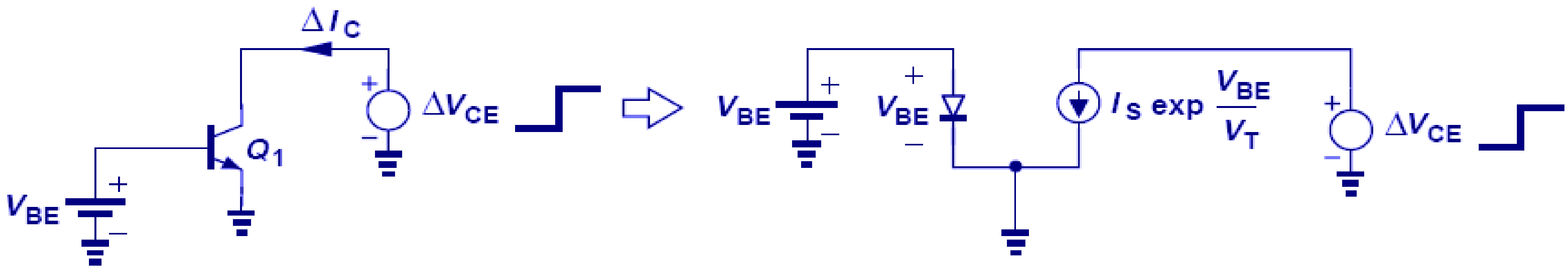
Small-Signal Model: V_{BE} Change

- Change in V_{BE} while V_{CE} is constant → $\Delta I_C = g_m \Delta V_{BE}$
- A voltage-controlled current source must be connected between the collector and the emitter with a value equal to $g_m \Delta V_{BE}$ (V_π is also used)
- Again, $\Delta I_B = \Delta I_C / \beta = (g_m/\beta)\Delta V_{BE}$ see next slide for the derivation
- By Ohm's Law, a resistor is placed between the base and emitter with the value of $r_\pi = \Delta V_{BE}/\Delta I_B = \beta/g_m$



Small-Signal Model: V_{CE} Change

- Ideally, V_{CE} has no effect on I_C or I_B. Thus, it will not contribute to the small signal model. (V_{CE} does really affect on I_C but can be neglected here)
- It can be shown that V_{CB} has no effect on the small signal model, either.



$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{1}{V_T} I_S e^{\frac{V_{BE}}{V_T}} = \frac{I_C}{V_T}$$

$$I_C = \beta \cdot I_B \Rightarrow \partial I_C = \beta \cdot \partial I_B$$

$$\partial I_C = \frac{I_C}{V_T} \partial V_{BE}$$

$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\beta \cdot V_T}{I_C}$$

AC Ground 交流接地

- Since the power supply voltage does not vary with time, it is regarded as a ground in small-signal analysis.
- Similarly, any other constant voltage in the circuit is replaced with a ground connection.
- To emphasize that such grounding holds for only signals, we sometimes say a node is an "***ac ground***."

Example 4.10

Consider the circuit shown in Fig. 4.24(a), where v_1 represents the signal generated by a microphone, $I_S = 3\text{x}10^{-16}$ A, $\beta = 100$, and Q_1 operates in the active mode. (a) If $v_1 = 0$, determine the small-signal parameters of Q_1. (b) If the microphone generates a 1-mV signal, how much change is observed in the collector and base currents?

Solution

(a) Writing $I_C = I_S \exp(V_{BE}/V_T)$, we obtain a collector bias current of 6.92 mA for $V_{BE} = 800\text{mV}$. Thus, $g_m = I_C/V_T = 1/3.75\Omega$, and $r_\pi = \beta/g_m = 375\Omega$.

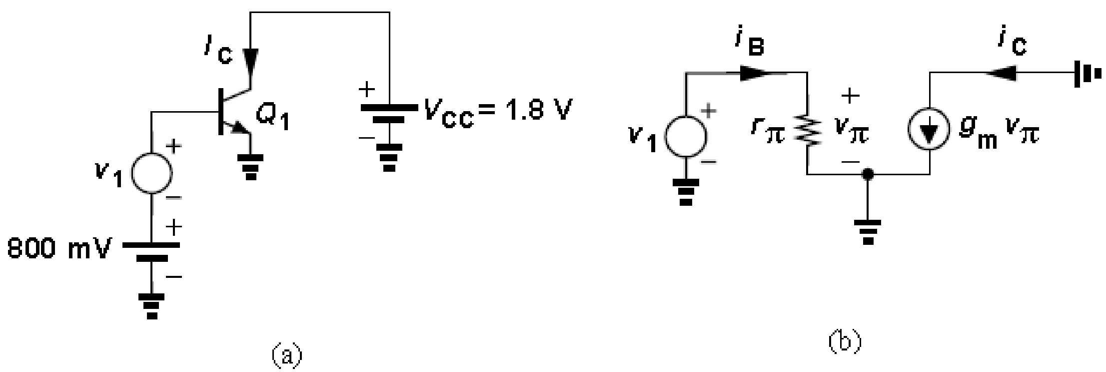
(b) Drawing the small-signal equivalent of the circuit as shown in Fig. 4.24(b) and recognizing that $v_\pi = v_1$, we obtain the change in the collector current as:

$\Delta I_C = g_m v_1 = 1\text{mV}/3.75\Omega = 0.267\text{mA}$.

The equivalent circuit also predicts the change in the base current as

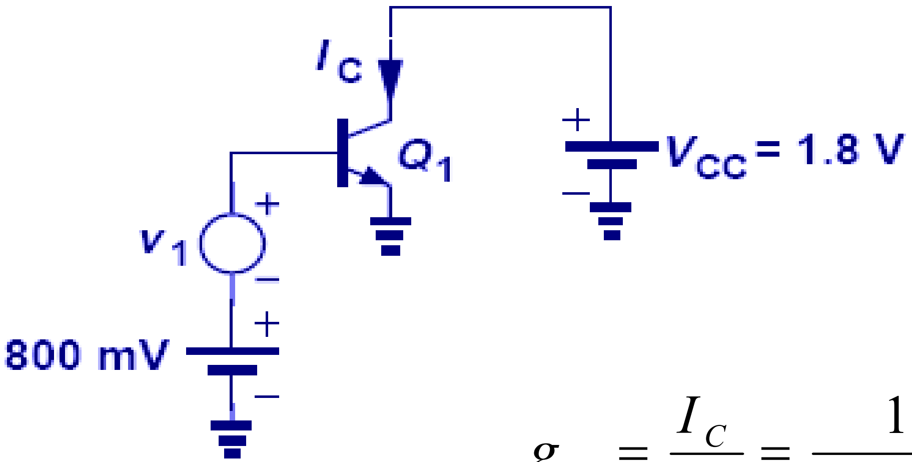
$\Delta I_B = v_1/r_\pi = 1\text{mV}/375\Omega = 2.67\mu\text{A}$.

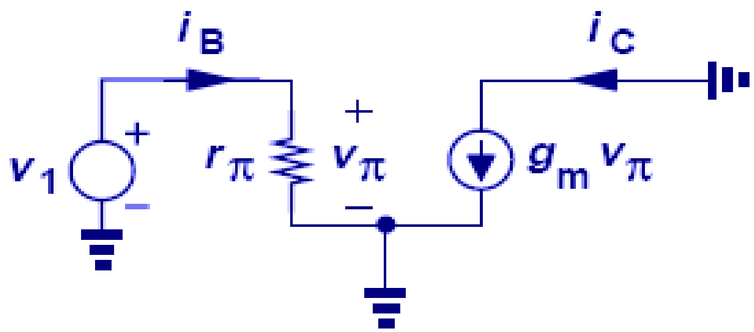
which is, of course, equal to $\Delta I_C/\beta$.



Example 4.10

- Here, small signal parameters are calculated from DC operating point and are used to calculate the change in collector current due to a change in V_{BE}.
- It is not a useful circuit. The microphone signal produces a change in I_C, but the result flows through the 1.8-V battery. In other words, the circuit generates no output.



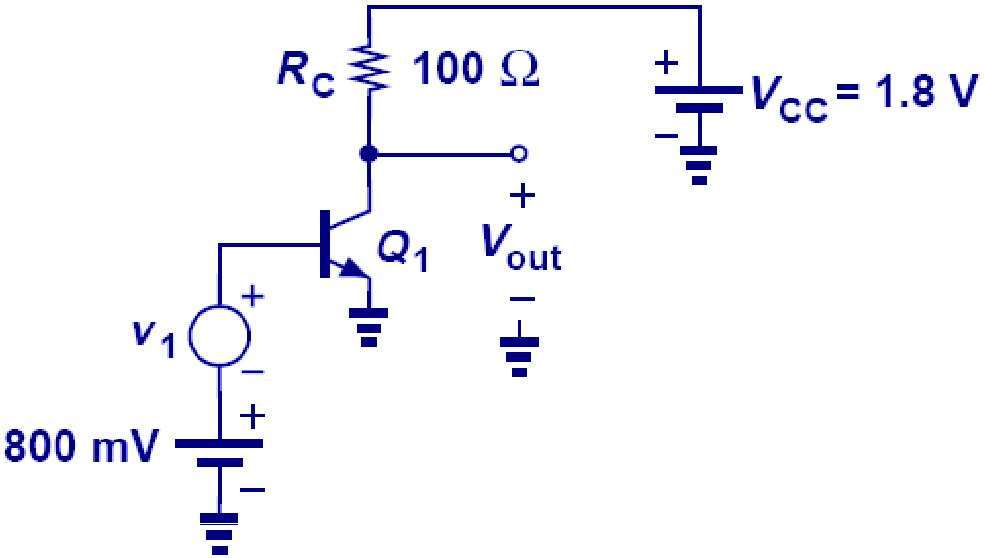


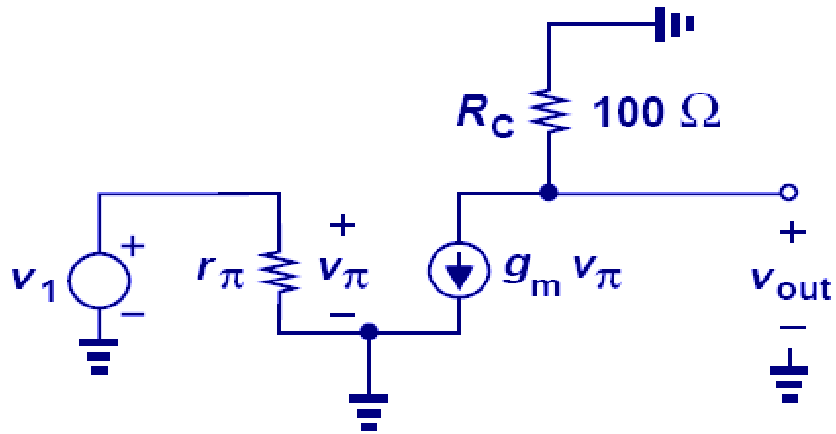
$$g_m = \frac{I_C}{V_T} = \frac{1}{3.75\,\Omega} = 0.267$$

$$r_\pi = \frac{\beta}{g_m} = 375\ \Omega$$

Small Signal Example II

- In this example, a resistor is placed between the power supply and collector, therefore, providing an output voltage.





Example 4.11

The circuit of Fig. 4.24(a) is modified as shown in Fig. 4.25, where resistor R_C converts the collector current to a voltage. (a) Verify that the transistor operates in the active mode. (b) Determine the output signal level if the microphone produces a 1-mV signal.

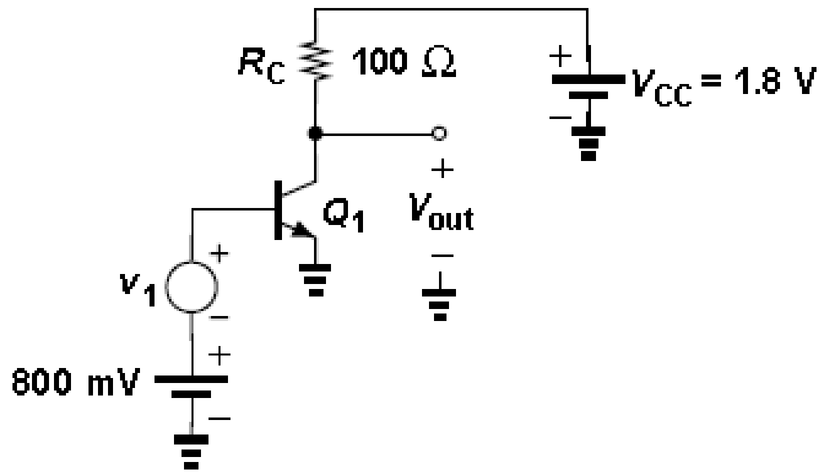
Solution

(a) The collector bias current of 6.92 mA flows through R_C, leading to a potential drop of 692 mV. The collector voltage, which is equal to V_{out}, is thus given by:

$$V_{out} = V_{CC} - I_C R_C = 1.108\text{ V}.$$

Since the collector voltage (with respect to ground) is more positive than the base voltage, the device operates in the active mode.

(b) As seen in the previous example, a 1-mV microphone signal leads to a 0.267-mA change in I_C. Upon flowing through R_C, this change yields a change of 0.267mA x 100Ω = 26.7 mV in V_{out}. The circuit therefore *amplifies* the input by a factor of 26.7.



Example 4.12

Considering the previous Example, suppose we raise R_C to 200Ω and V_{CC} to 3.6 V. Verify that the device operates in the active mode and compute the voltage gain.

Solution

The voltage drop across R_C now increases to 6.29mA x 200Ω = 1.384V, leading to a collector voltage of 3.6 - 1.384 = 2.216 V and guaranteeing operation in the active mode. Note that if is not doubled, then V_{out} = 1.8 - 1.384 = 0.416V and the transistor is not in the forward active region.

\

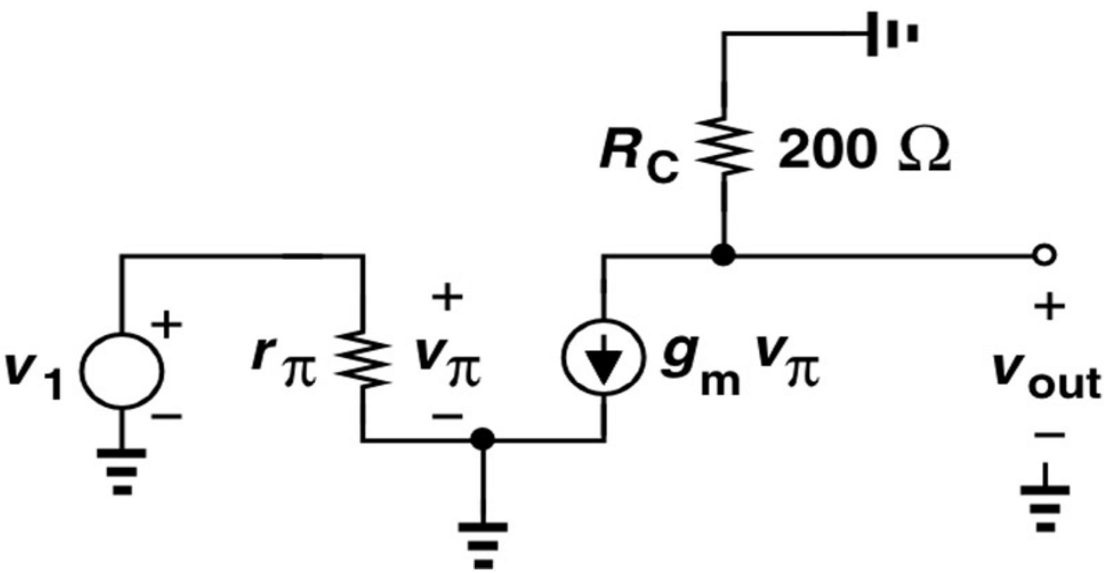


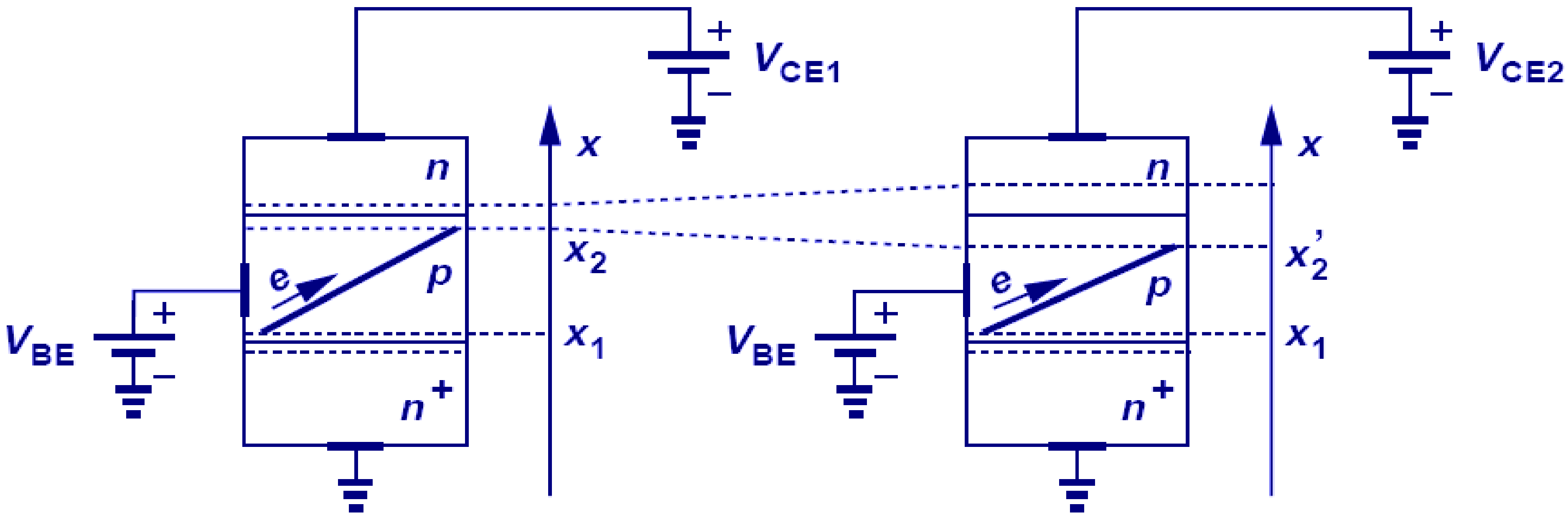
Fig. 4.26 Small-signal equivalent circuit of the stage shown in Fig. 4.25.

Recall from part (b) of the above example that the change in the output voltage is equal to the change in the collector current multiplied by R_C. Since R_C is doubled, the voltage gain must also double, reaching a value of 53.4. This result is also obtained with the aid of the small-signal model. Illustrated in Fig. 4.26, the equivalent circuit yields $v_{out} = -g_m v_\pi R_C = -g_m v_1 R_C$ and hence $v_{out}/v_1 = -g_m R_C$. With $g_m = (3.75\Omega)^{-1}$ and $R_C = 200\Omega$, we have $v_{out}/v_1 = -53.4$.

- This example points to an important trend: if R_C increases, so does the voltage gain of the circuit.
- Does this mean that, if $R_C \rightarrow \infty$, then the gain also grows indefinitely?
- Indeed, the "***Early effect***" translates to a nonideality in the device that can limit the gain of amplifiers.

Early Effect

- The claim that collector current does not depend on V_{CE} is not accurate.
- As V_{CE} increases, the depletion region between base and collector increases. Therefore, the effective base width decreases, which leads to an increase in the collector current.



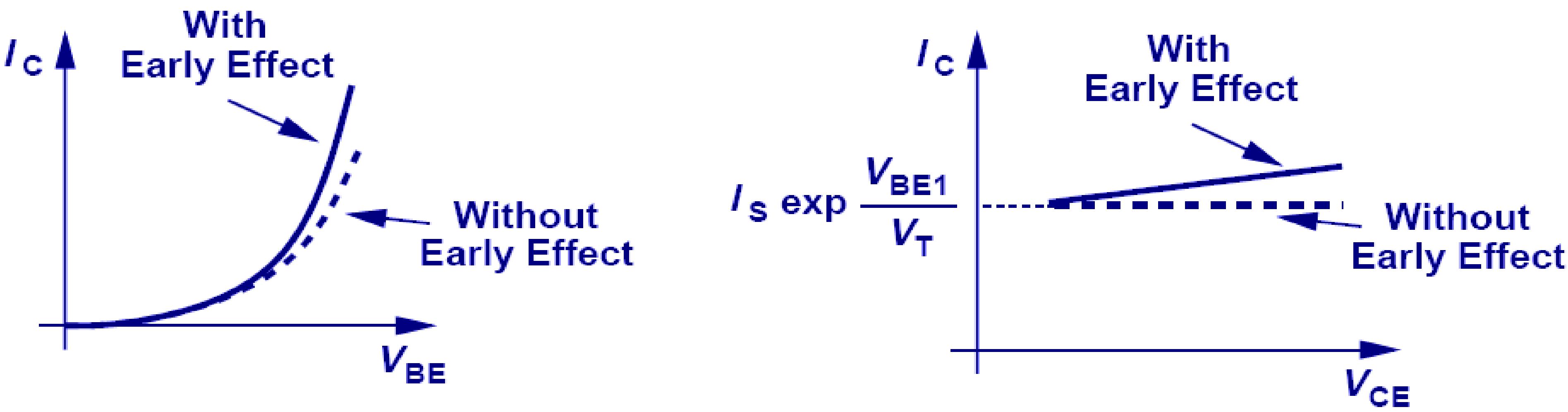
Early Effect Illustration

- With Early effect, I_C becomes larger than usual and a function of V_{CE}.
- I_C can be approximately expressed by

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(\exp\frac{V_{BE}}{V_T} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right), \approx \left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE}}{V_A}\right).$$

  where W_B is assumed constant and the second factor, $1 + V_{CE}/V_A$, models the Early effect.

- V_A is called the "***Early voltage***."



(a) Fig. 4.28 (b)

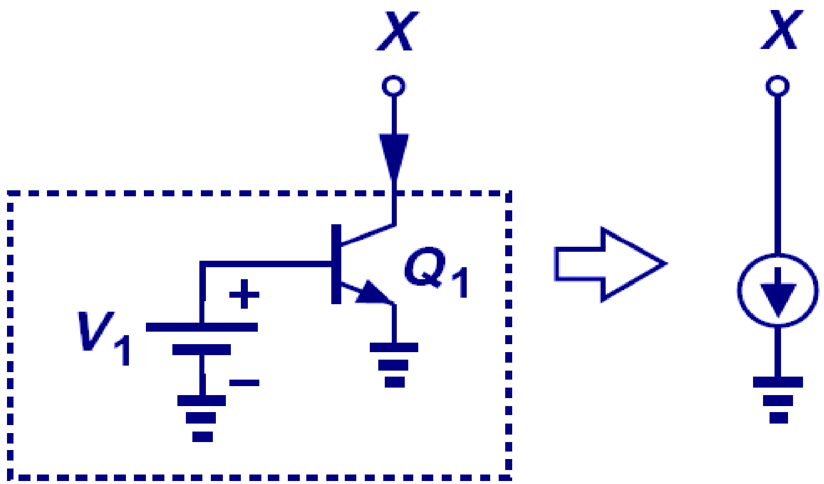
- For a constant V_{CE}, the dependence of I_C upon V_{BE} remains exponential but with a somewhat greater slope [Fig. 4.28(a)].
- For a constant V_{BE}, the I_C - V_{CE} characteristic displays a nonzero slope [Fig. 4.28(b)].
- Differentiation of I_C with respect to V_{CE} yields

$$\frac{\delta I_C}{\delta V_{CE}} = I_S \left(\exp \frac{V_{BE}}{V_T} \right) \left(\frac{1}{V_A} \right) \approx \frac{I_C}{V_A},$$

where it is assumed $V_{CE} << V_A$ and hence $I_C \approx I_S \exp(V_{BE}/V_T)$

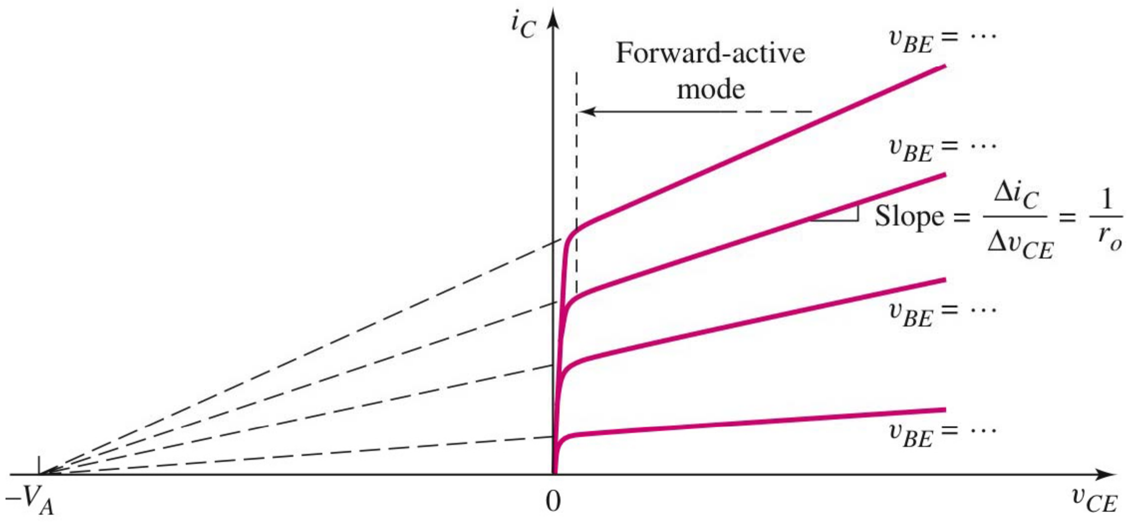
Early Effect Representation

- *non-ideal* current source



$$V_X \rightarrow V_{CE}$$

$$\left(I_S \exp\frac{V_1}{V_T}\right)\left(1+\frac{V_X}{V_A}\right)$$



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Example 4.13

A bipolar transistor carries a collector current of 1 mA with $V_{CE} = 2$ V. Determine the required base-emitter voltage if $V_A = \infty$ or $V_A = 20$V. Assume $I_S = 2\text{x}10^{-16}$ A.

Solution

With $V_A = \infty$, we have

$$V_{BE} = V_T \ln \frac{I_C}{I_S} = 760.3\text{mV}$$

If $V_A = 20$V, we have

$$V_{BE} = V_T \ln \left(\frac{I_C}{I_S} \frac{1}{1 + \frac{V_{CE}}{V_A}} \right) = 757.8\text{mV}.$$

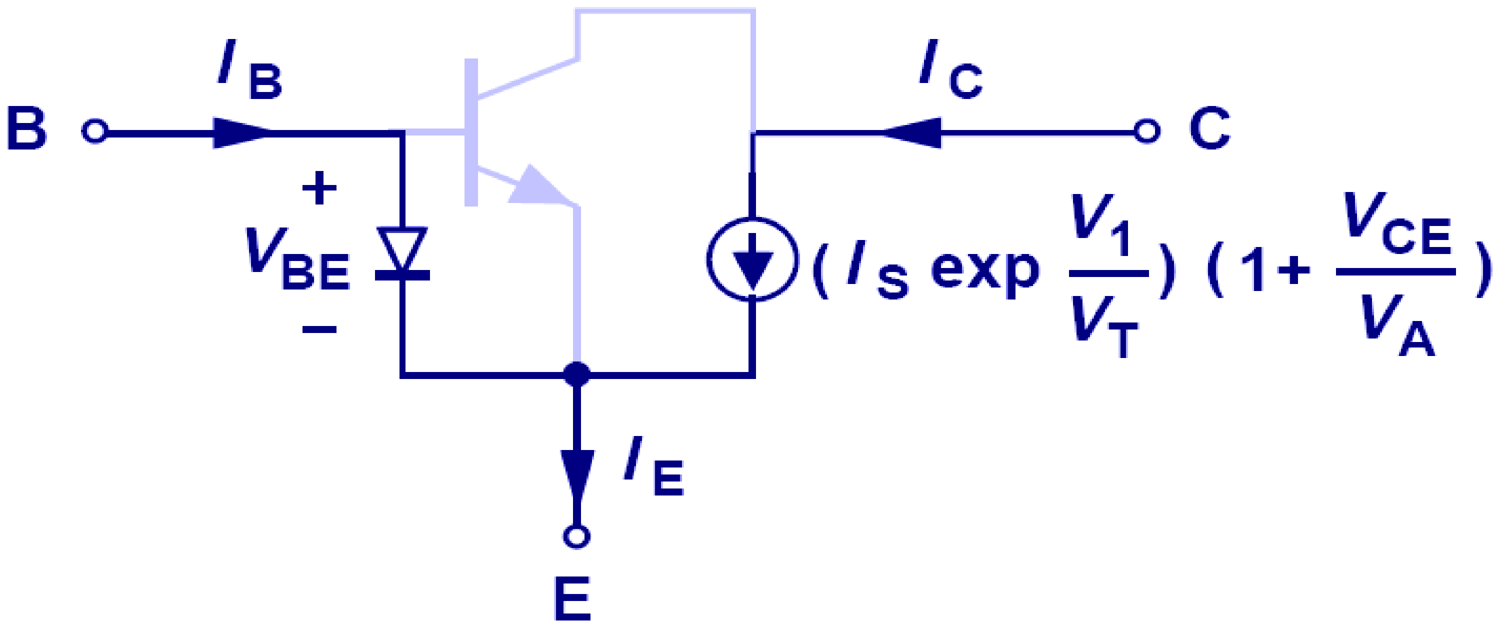
In fact, for $V_{CE} << V_A$, we have $(1 + V_{CE}/V_A)^{-1} = 1 - V_{CE}/V_A$

$$V_{BE} \approx V_T \ln \frac{I_C}{I_S} + V_T \ln \left(1 - \frac{V_{CE}}{V_A} \right) \approx V_T \ln \frac{I_C}{I_S} - V_T \frac{V_{CE}}{V_A},$$

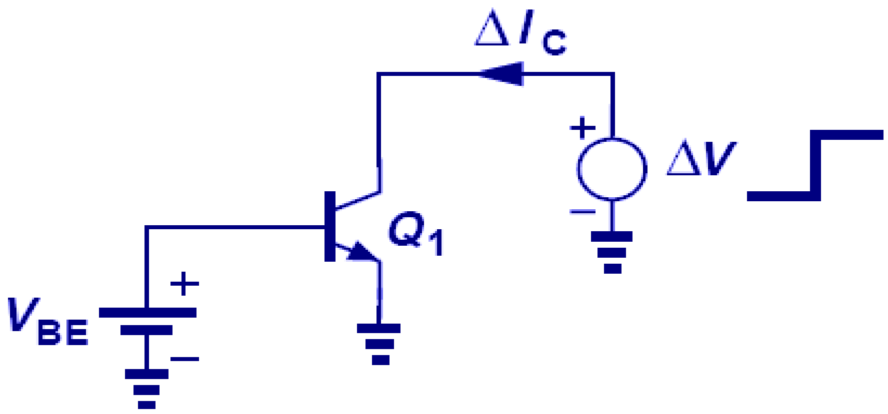
where it is assumed $\ln(1 - \epsilon) \approx -\epsilon$ for $\epsilon << 1$.

Early Effect and Large-Signal Model

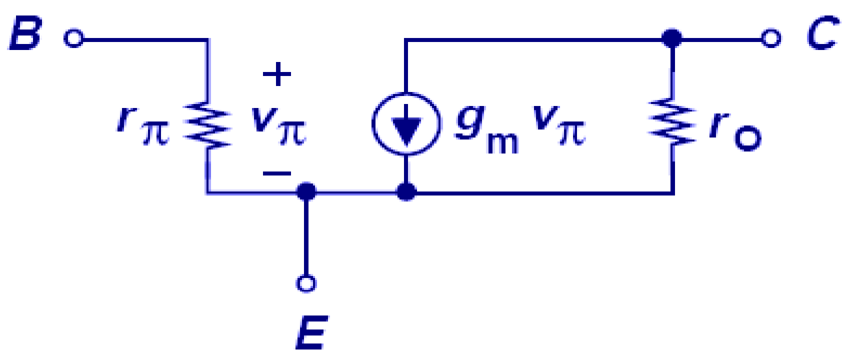
- Early effect can be accounted for in large-signal model by simply changing the collector current with a correction factor, $1 + V_{CE}/V_A$.
- In this mode, base current does not change.



Early Effect and Small-Signal Model



(a)



(b)

$$g_m = \frac{dI_C}{dV_{BE}} = \frac{1}{V_T}\left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$

$$= \frac{I_C}{V_T}.$$

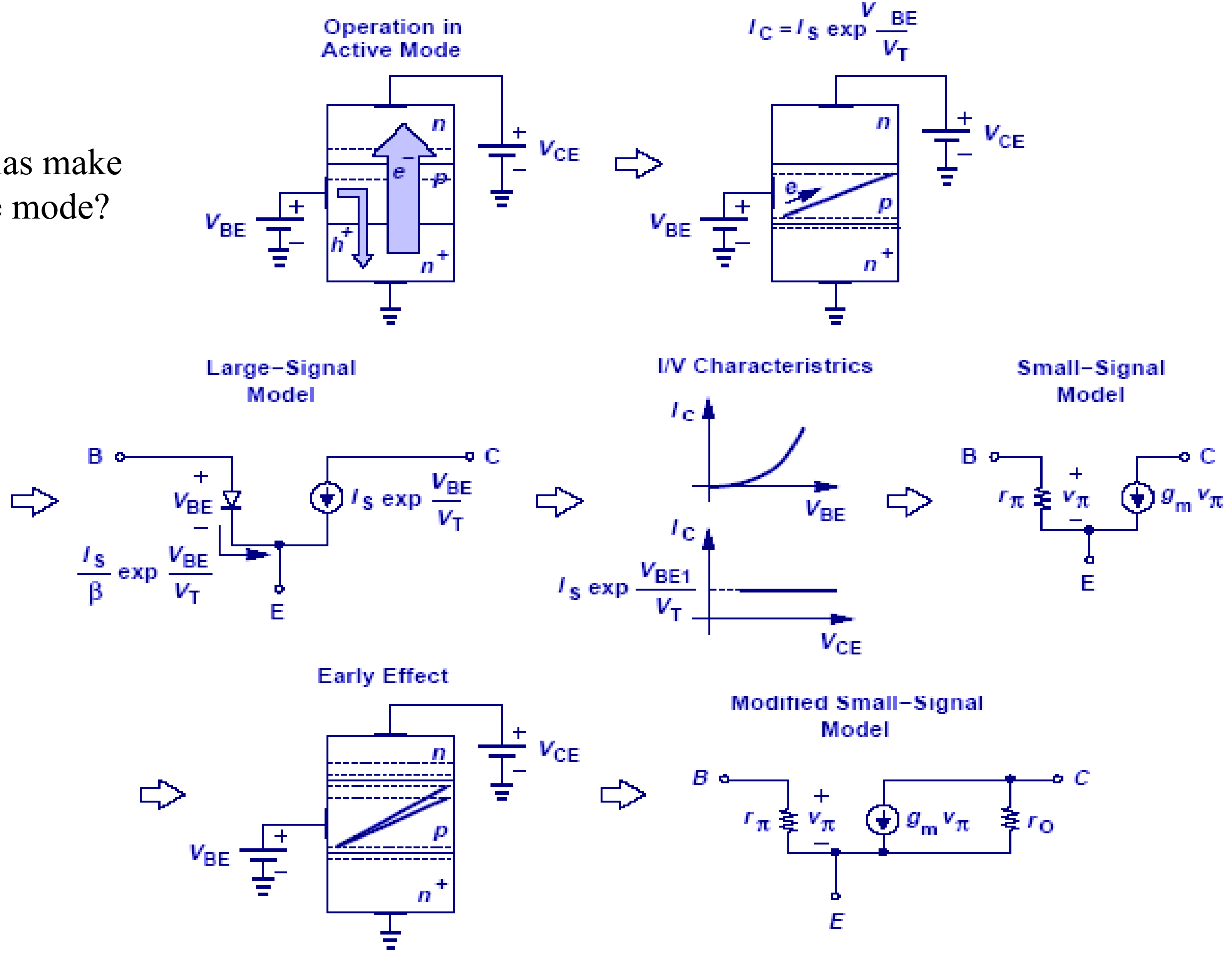
$$r_\pi = \frac{\beta}{g_m} = \beta\frac{V_T}{I_C}.$$

$$r_o = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{V_A}{I_S \exp\frac{V_{BE}}{V_T}} \approx \frac{V_A}{I_C}$$

- One extra element, r_O, to represent the Early effect.
- Called the "***output resistance,***" plays a critical role in high-gain amplifiers

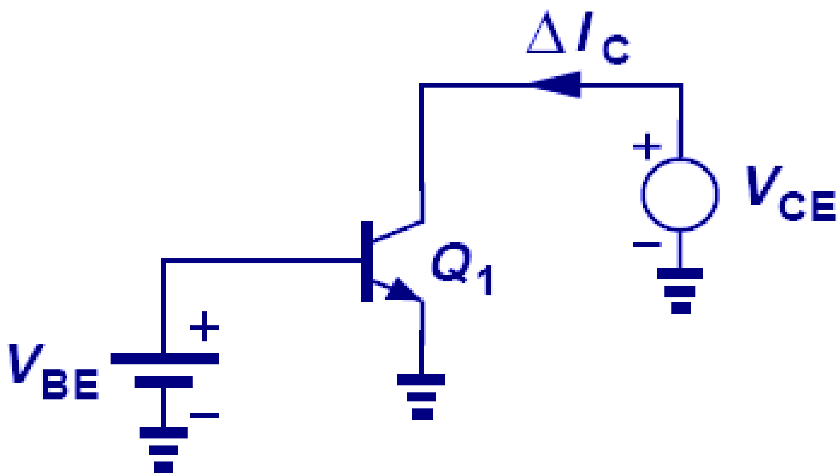
Summary of Active Mode

Recall:
What bias make
it active mode?

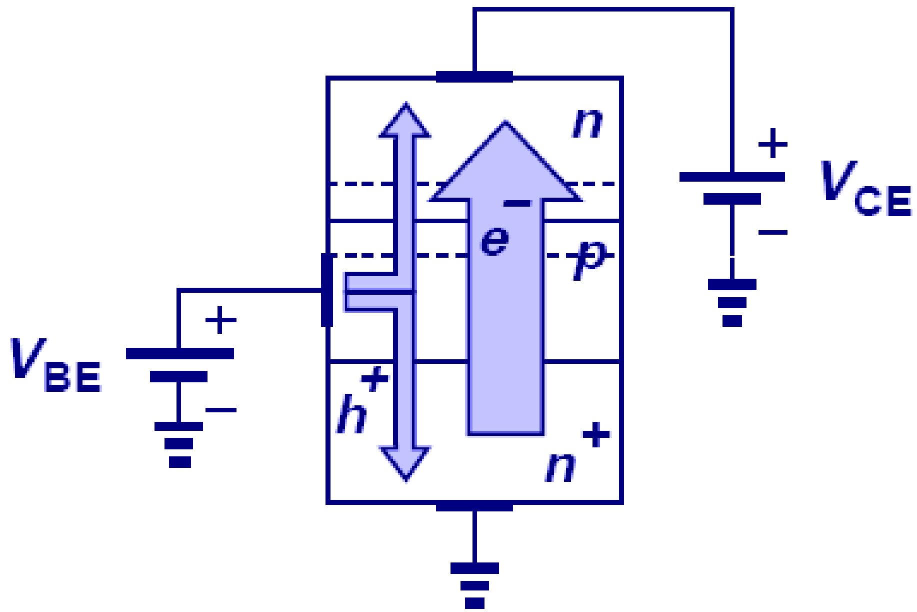


Bipolar Transistor in Saturation

- When collector voltage drops below base voltage and forward biases the collector-base junction, base current increases and decreases the current gain factor, β.
- $V_{BE} > V_{CE}$, both junctions are forward bias, transistor is in "***saturation***."



(a)



(b)

Example 4.15

A bipolar transistor is biased with V_{BE} = 750 mV and has a nominal β of 100. How much B-C forward bias can the device tolerate if β must not degrade by more than 10%? For simplicity, assume base-collector and base-emitter junctions have identical structures and doping levels.

Solution

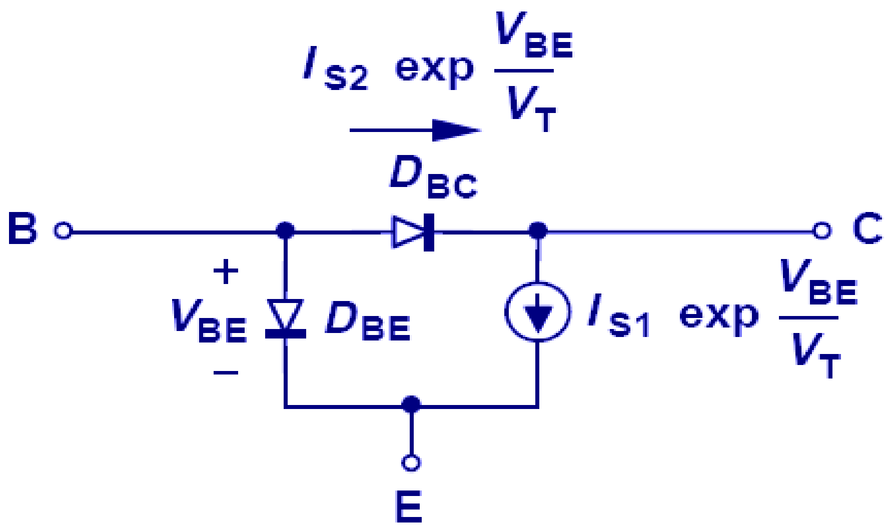
If the base-collector junction is forward-biased so much that it carries a current equal to one-tenth of the nominal base current, I_B, then the β degrades by 10%. Since $I_B = I_C/100$, the B-C junction must carry no more than $I_B = I_C/1000$. We therefore ask, what B-C voltage results in a current of $I_C/1000$ if V_{BE} = 750 mV gives a collector current of I_C? Assuming identical B-E and B-C junctions, we have

$$V_{BE} - V_{BC} = V_T \ln \frac{I_C}{I_S} - V_T \ln \frac{I_C/1000}{I_S} = V_T \ln 1000 \approx 180\text{mV}.$$

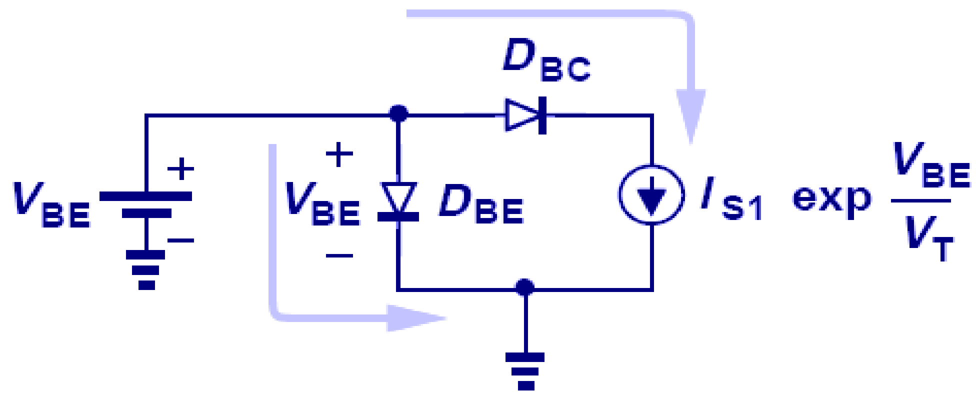
That is, V_{BC} = 750 - 180 = 570 mV.

this gives V_{CE} = 180 mV

Large-Signal Model for Saturation Region



(a)

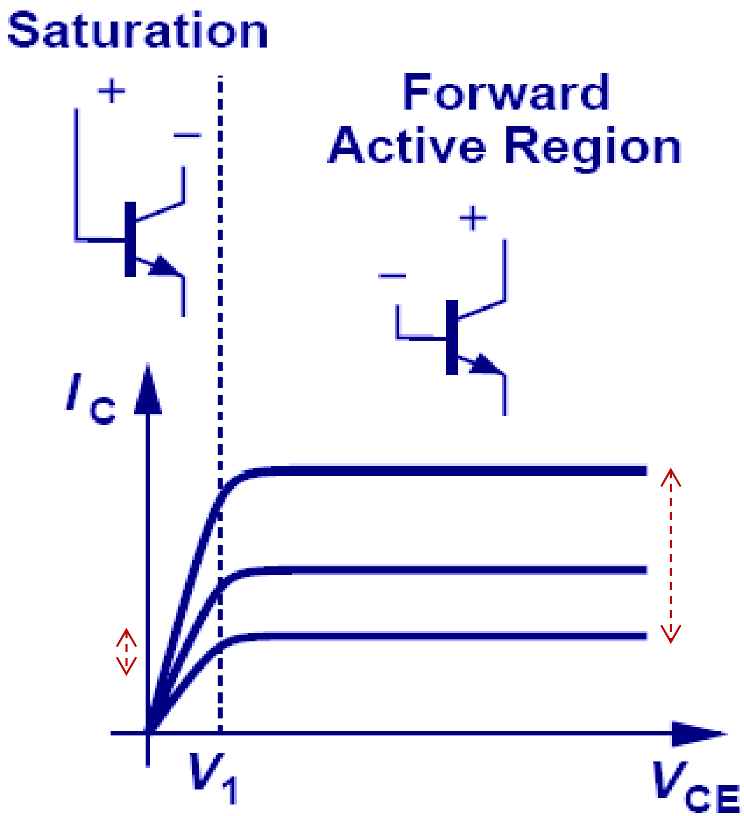


(b)

if collector is open

Overall I/V Characteristics

- Increasing I_B in the ***saturation*** region leads to little change in I_C β↓
- The speed of the BJT also drops in saturation.
- As a rule of thumb, we permit *soft* saturation with V_{BC} < 400 mV because the current in the B-C junction is negligible.
- That is, $V_{CE} = V_{BE} - V_{BC} \approx 300\sim400$ mV



Example 4.16

For the circuit of Fig. 4.36, determine the relationship between R_C and V_{CC} that guarantees operation in soft saturation or active region.

Solution

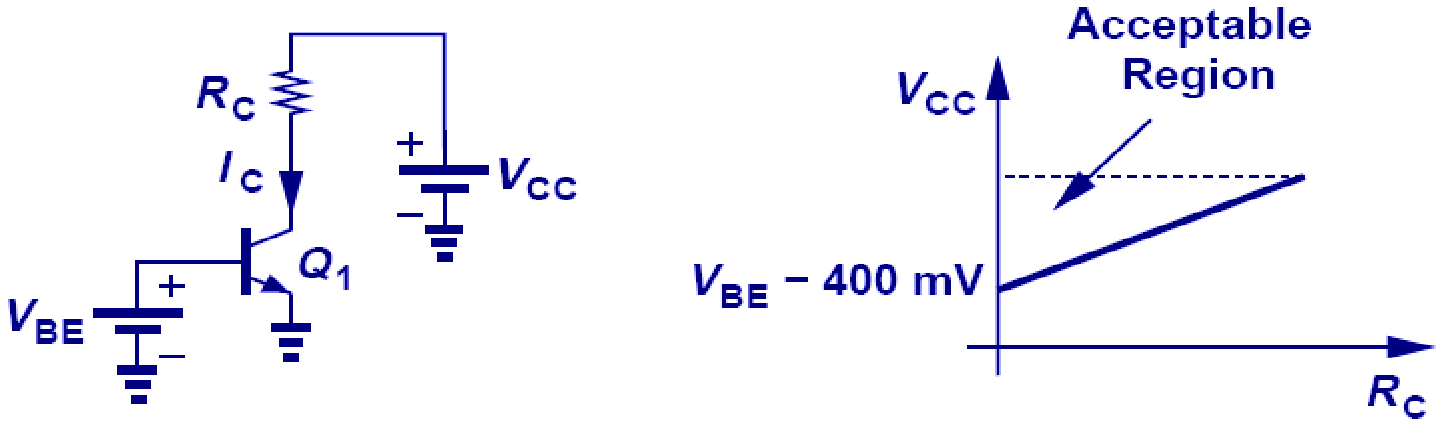
In soft saturation, the collector current is still equal to $I_S \exp(V_{BE}/V_T)$. The collector voltage must not fall below the base voltage by more than 400 mV:

$$V_{CC} - I_C R_C \geqq V_{BE} - 400 \text{ mV}$$

Thus,

$$V_{CC} \geqq I_C R_C + V_{BE} - 400 \text{ mV}$$

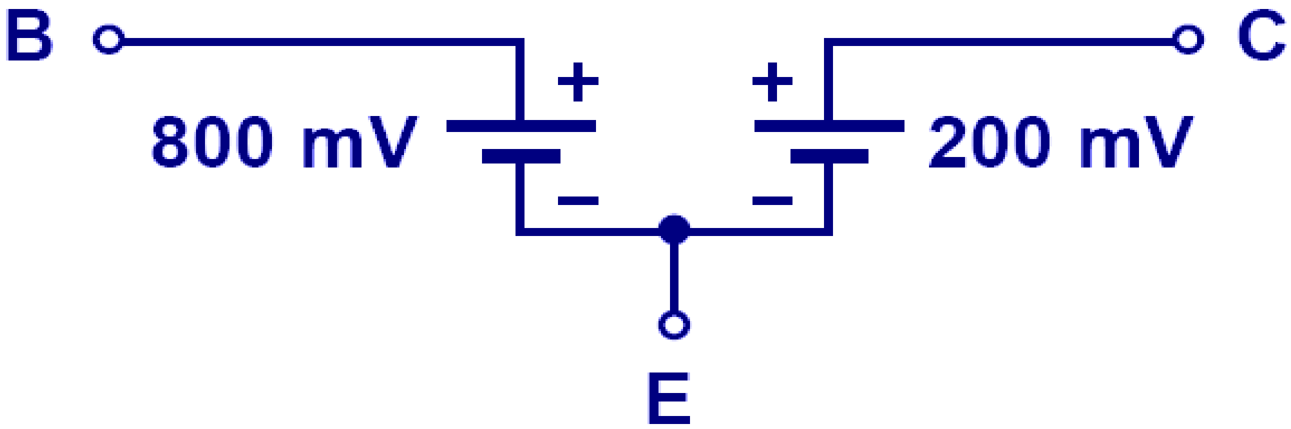
For a given value of R_C, V_{CC} must be sufficiently large so that $V_{CC} - I_C R_C$ still maintains a reasonable collector voltage.



(a) (b)

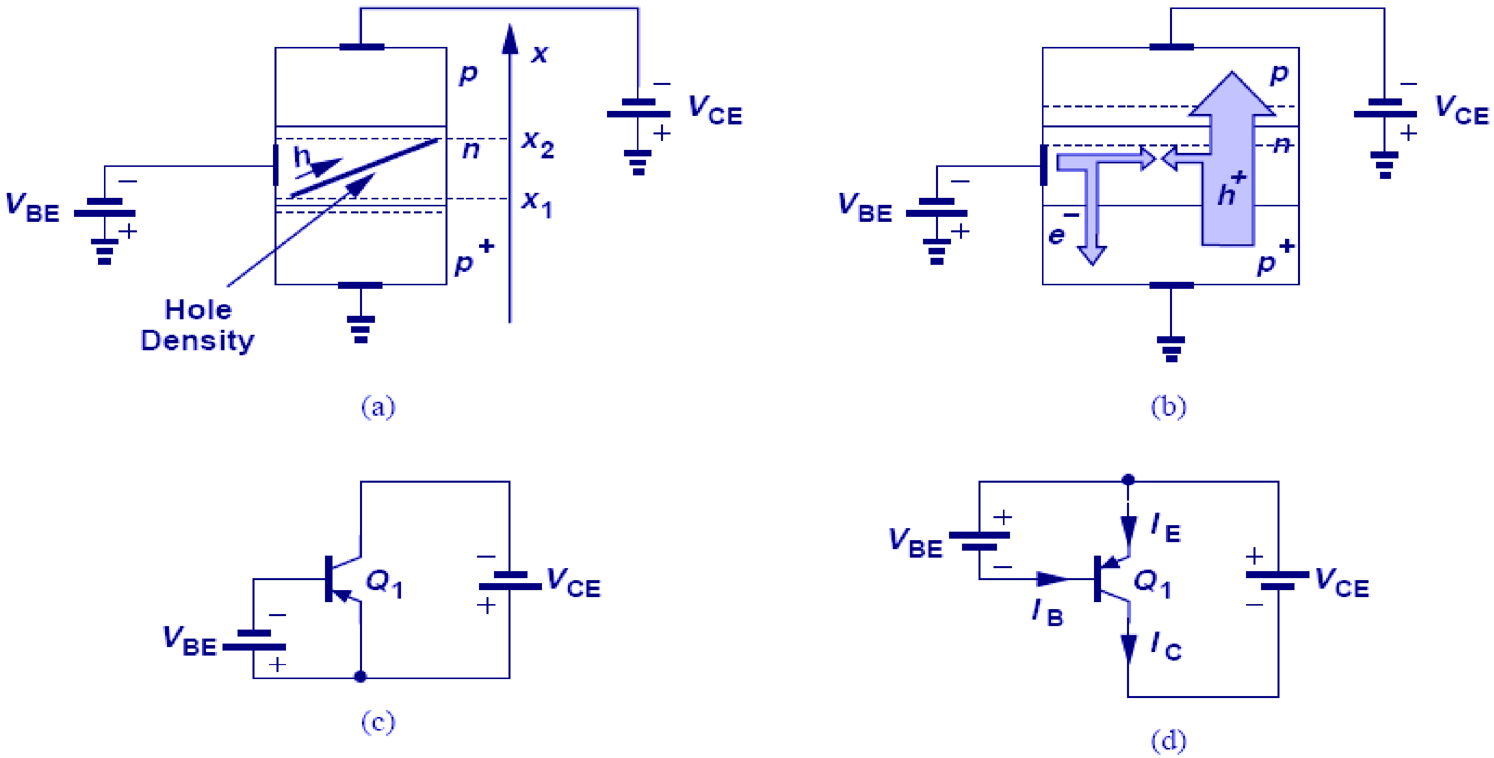
Deep Saturation

- In deep saturation region, the transistor loses its voltage-controlled current capability and V_{CE} approaches a constant value called $V_{CE,sat}$ (about 200 mV)



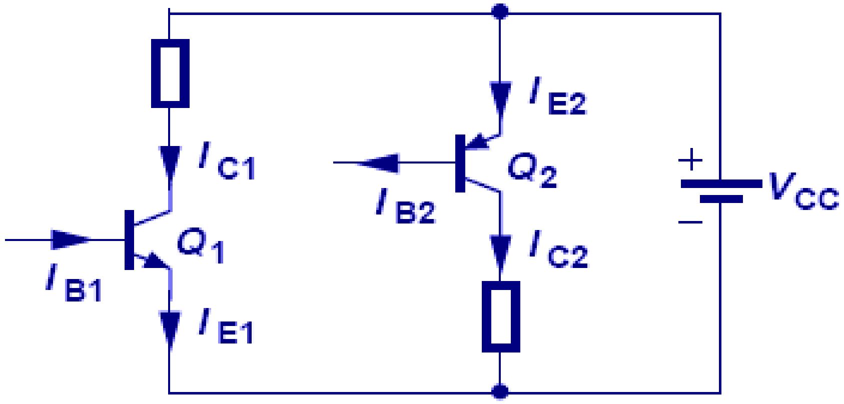
PNP Transistor

- With the polarities of emitter, collector, and base reversed, a PNP transistor is formed.
- All the principles that applied to NPN's also apply to PNP's, with the exception that emitter is at a higher potential than base and base at a higher potential than collector.

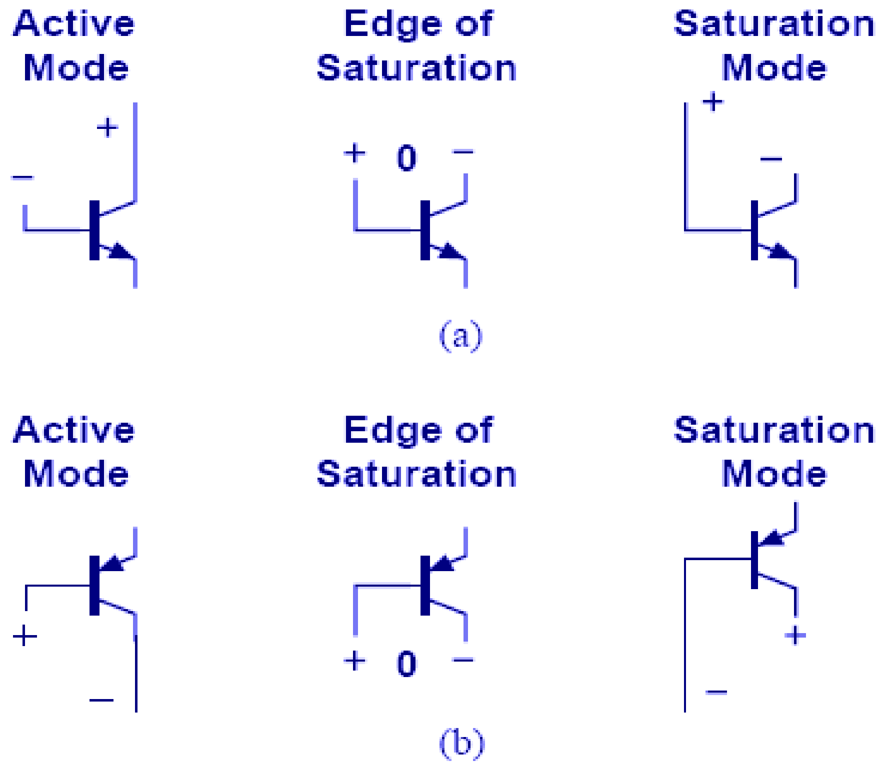


A Comparison between NPN and PNP Transistors

- The figure below summarizes the direction of current flow and operation regions for both the NPN and PNP BJT's.



(a)



PNP Equations

$$I_C = I_S \exp \frac{V_{EB}}{V_T}$$

$$I_B = \frac{I_S}{\beta} \exp \frac{V_{EB}}{V_T}$$

$$I_E = \frac{\beta + 1}{\beta} I_S \exp \frac{V_{EB}}{V_T}$$

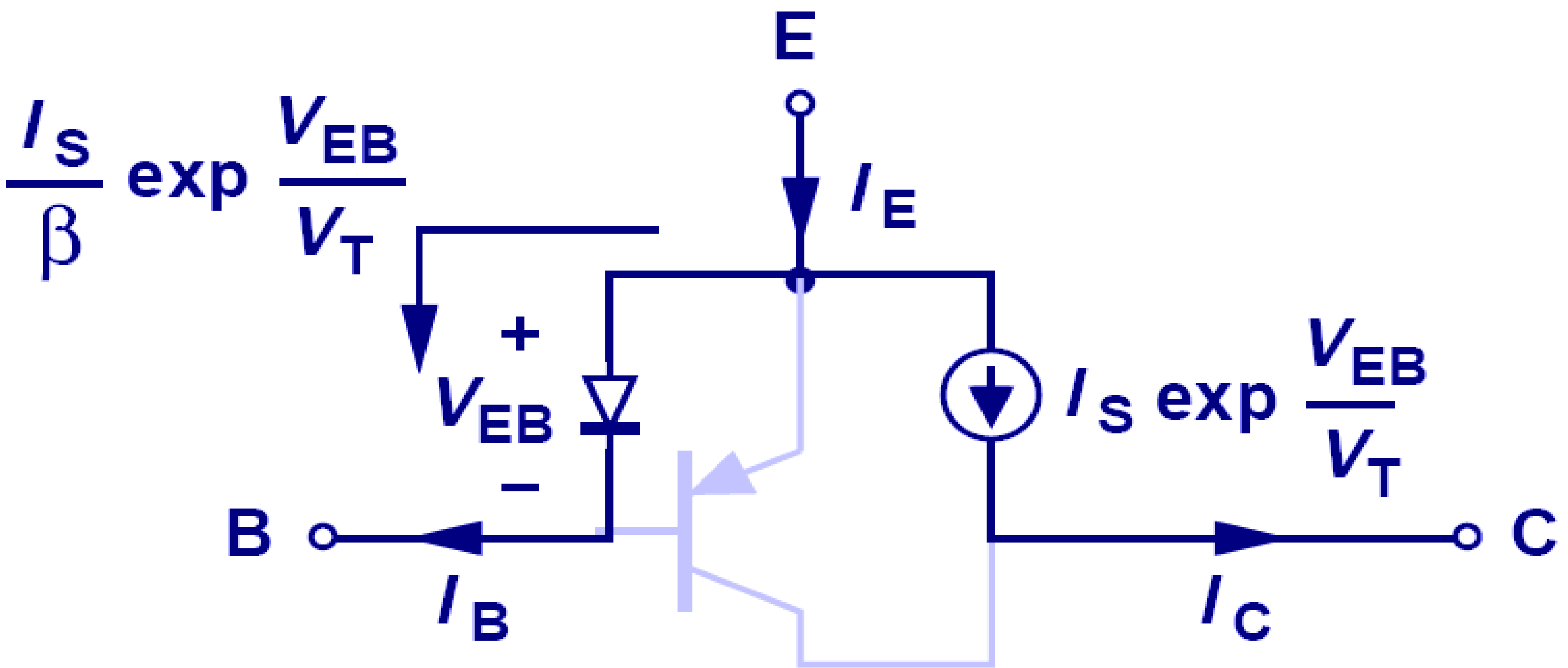
Early Effect ⇒

$$I_C = \left(I_S \exp \frac{V_{EB}}{V_T} \right) \left(1 + \frac{V_{EC}}{V_A} \right)$$

➢ Summary of the bipolar current-voltage relationships in the active region.

npn	pnp
$i_C = I_S e^{\frac{v_{BE}}{V_T}}$	$i_C = I_S e^{\frac{v_{EB}}{V_T}}$
$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha} e^{\frac{v_{BE}}{V_T}}$	$i_E = \frac{i_C}{\alpha} = \frac{I_S}{\alpha} \cdot e^{\frac{v_{EB}}{V_T}}$
$i_B = \frac{i_C}{\beta_F} = \frac{I_S}{\beta_F} e^{\frac{v_{BE}}{V_T}}$	$i_B = \frac{i_C}{\beta_F} = \frac{I_S}{\beta_F} \cdot e^{\frac{v_{EB}}{V_T}}$
For both transistors	
$i_E = i_C + i_B$	$i_C = \beta i_B$
$i_E = (1 + \beta_F) i_B$	$i_C = \alpha i_E = \left(\frac{\beta}{1+\beta}\right) i_E$
$\alpha_F = \frac{\beta_F}{1 + \beta_F}$	$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$

Large Signal Model for PNP



Example 4.17

In the circuit shown in Fig. 4.41, determine the terminal currents of Q_1 and verify operation in the forward active region. Assume $I_S = 2\text{x}10^{-16}$ A and $\beta = 50$, but $V_A = \infty$.

Solution

We have $V_{EB} = 2 - 1.2 = 0.8$ V and hence

$$I_C = I_S \exp\frac{V_{EB}}{V_T} = 4.61\text{mA}.$$

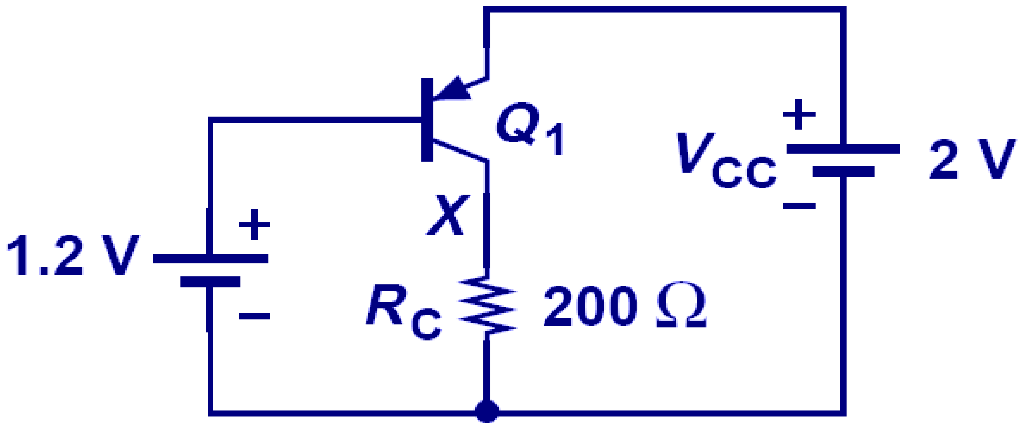
It follows that

$$I_B = 92.2\ \mu\text{A}, \quad I_E = 4.70\ \text{mA}$$

We must now compute the collector voltage and hence the bias across the B-C junction. Since R_C carries I_C,

$$V_X = R_C I_C = 0.922\ \text{V}$$

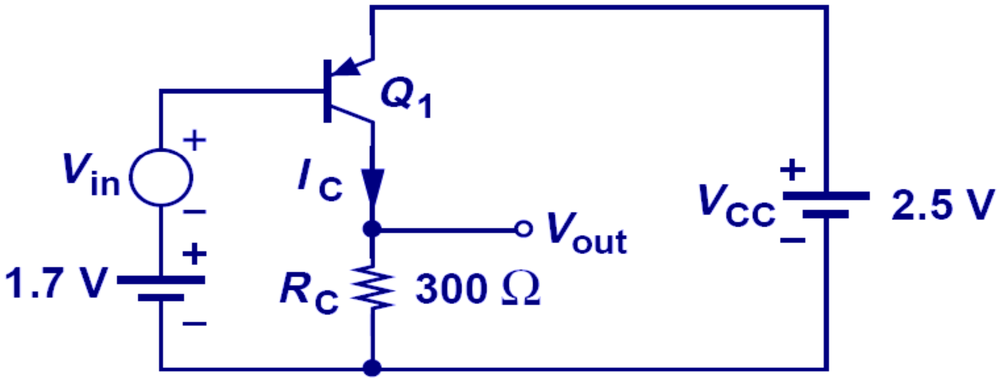
which is *lower* than the base voltage. Invoking the illustration in Fig. 4.39(b), we conclude that Q_1 operates in the active mode.



Example 4.18

In the circuit of Fig. 4.42, V_{in} represents a signal generated by a microphone. Determine V_{out} for $V_{in} = 0$ and $V_{in} = 5$ mV if $I_S = 1.5\text{x}10^{-16}$ A.

Solution



For $V_{in} = 0$, $V_{EB} = 800$ mV and we have

$$I_C\,|_{V_{in}=0} = I_S \exp\frac{V_{EB}}{V_T} = 3.46\text{mA},$$

and hence

$$V_{out} = 1.038 \text{ V}$$

If V_{in} increases to 5 mV, $V_{EB} = 795$ mV and

$$I_C\,|_{Vin=5\text{mV}} = 2.85 \text{ mA}$$

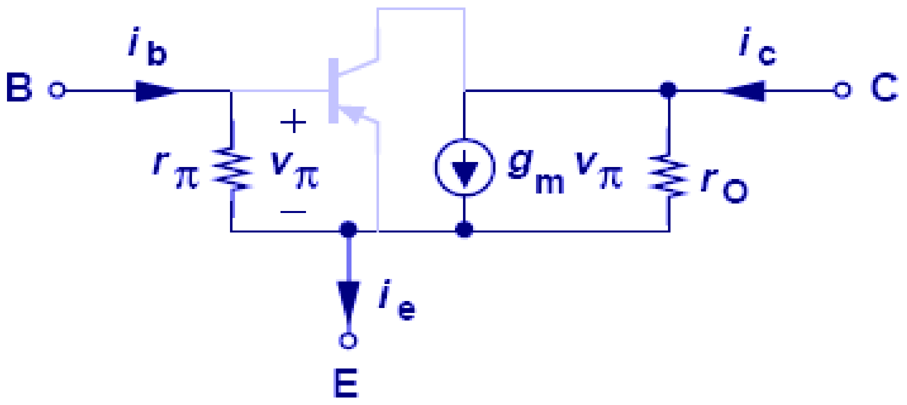
yielding

$$V_{out} = 0.856 \text{ V}$$

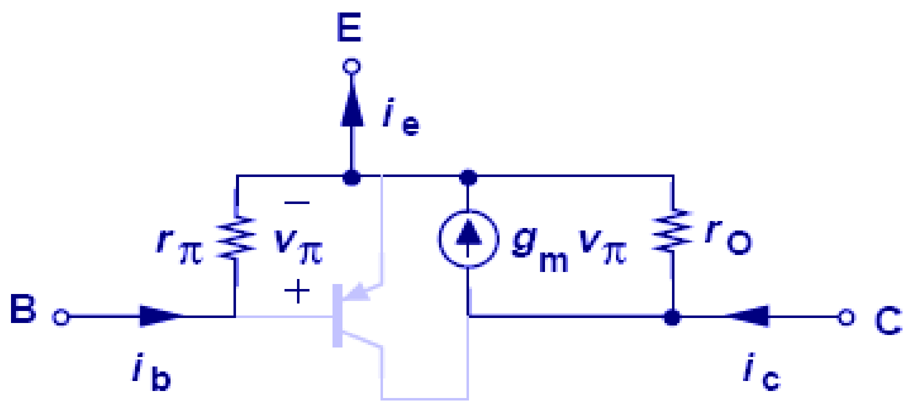
Note that as the base voltage *rises*, the collector voltage *falls*, a behavior similar to that of the *npn* counterparts in Figs. 4.25. Since a 5-mV change in V_{in} gives a 182-mV change in V_{out}, the voltage gain is equal to 36.4.These results are more readily obtained through the use of the small-signal model.

Small-Signal Model for PNP Transistor

- The small signal model for PNP transistor is exactly IDENTICAL to that of NPN. This is not a mistake because the current direction is taken care of by the polarity of V_{BE}.



(a)



(b)

Example 4.19

If the collector and base of a bipolar transistor are tied together, a two-terminal device results. Determine the small-signal impedance of the devices shown in Fig. 4.44(a). Assume $V_A = \infty$.

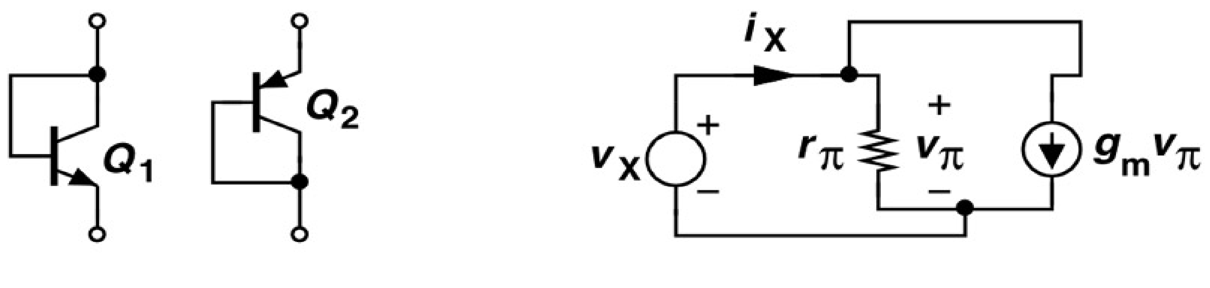
Solution

We replace the bipolar transistor Q_1 with its small-signal model and apply a small-signal voltage across the device [Fig. 4.44(b)]. Noting that r_π carries a current equal to v_X/r_π, we write a KCL at the input node:

$$\frac{v_X}{r_\pi} + g_m v_\pi = i_X.$$

Since $g_m r_\pi = \beta >> 1$, we have

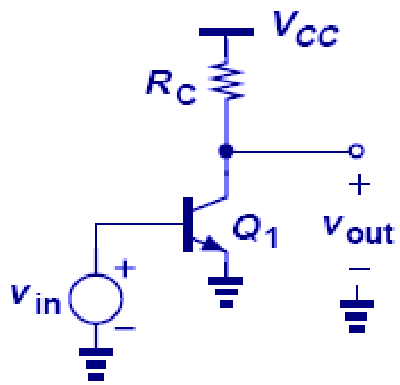
$$\frac{v_X}{i_X} = \frac{1}{g_m + r_\pi^{-1}} \approx \frac{1}{g_m} = \frac{V_T}{I_C}.$$

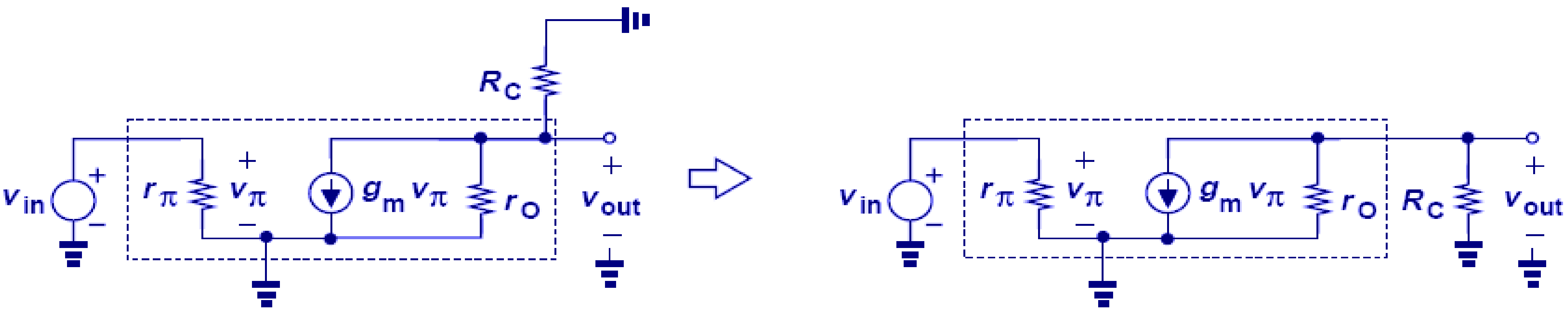


(a) (b)

Interestingly, with a bias current of I_C, the device exhibits an impedance similar to that of a diode carrying the same bias current. We call this structure a "***diode-connected transistor***." The same results apply to the *pnp* configuration in Fig. 4.44(a).

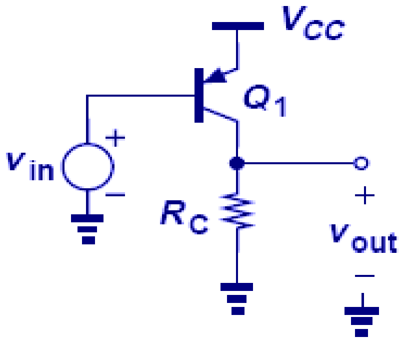
Small Signal Model Example I

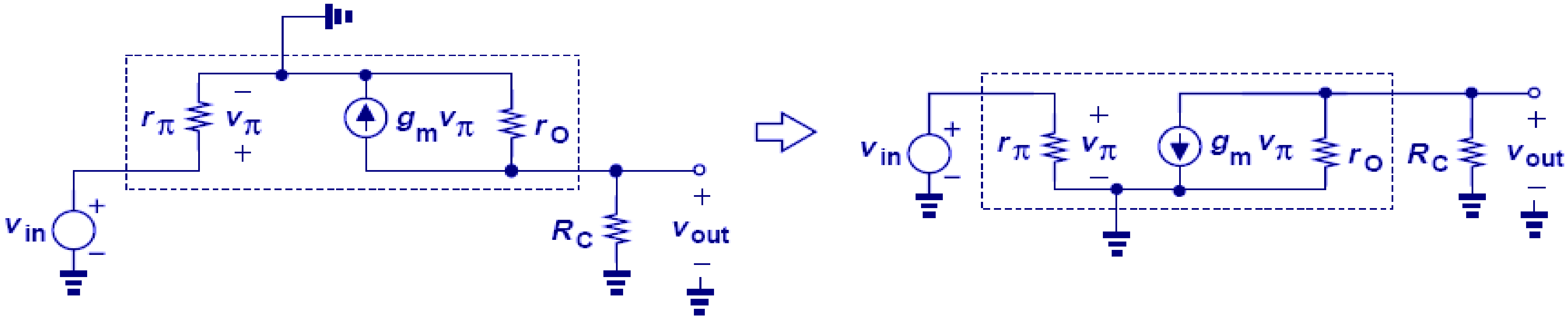




Small Signal Model Example II

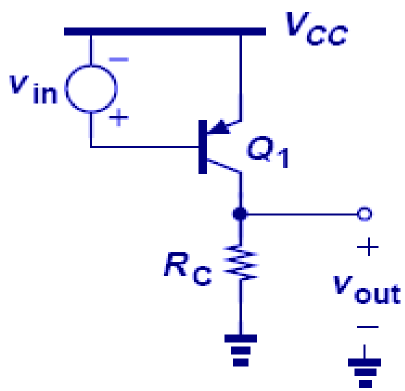
- Small-signal model is identical to the previous ones.



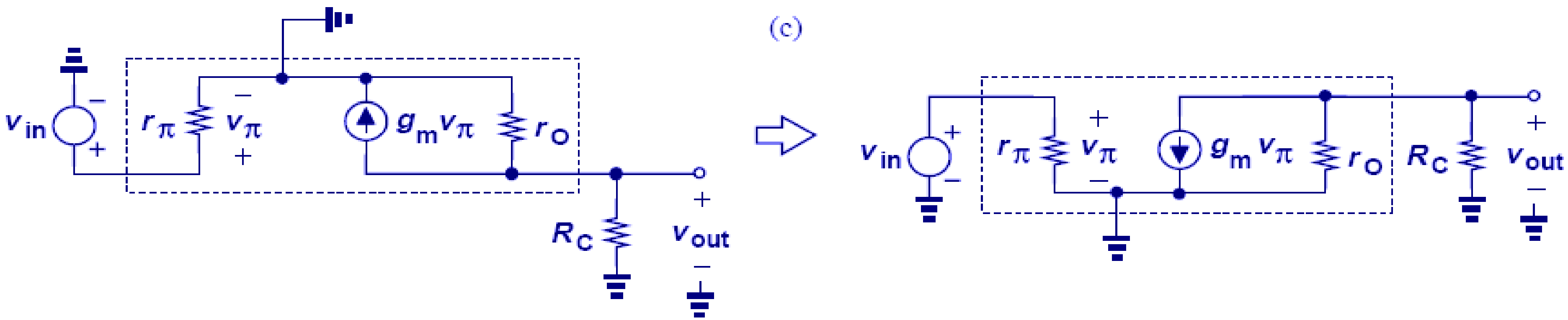


Small Signal Model Example III

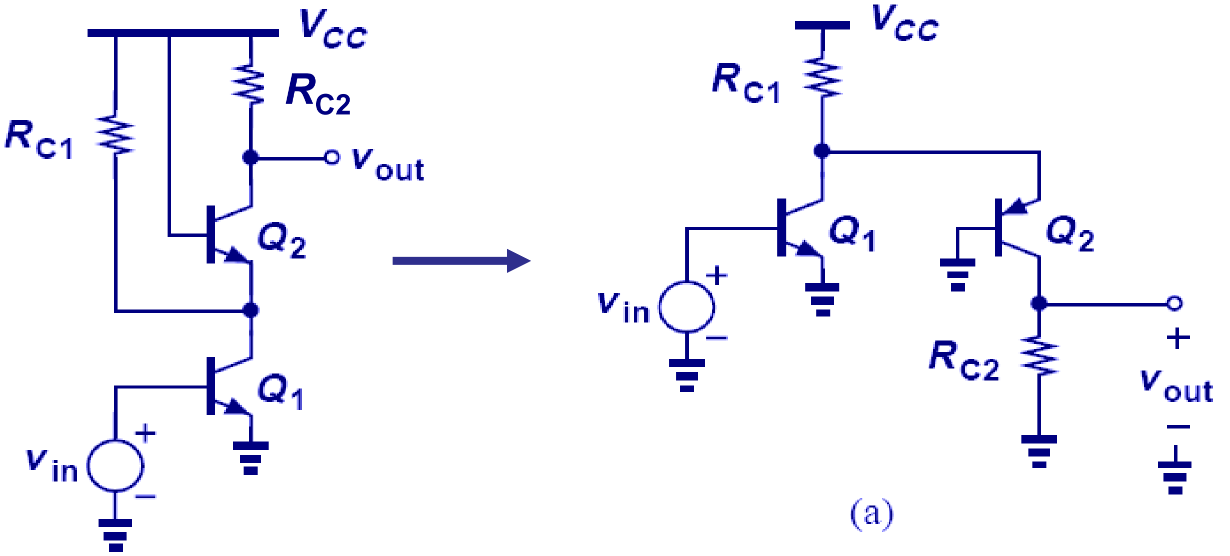
- Since during small-signal analysis, a constant voltage supply is considered to be AC ground, the final small-signal model is identical to the previous two.



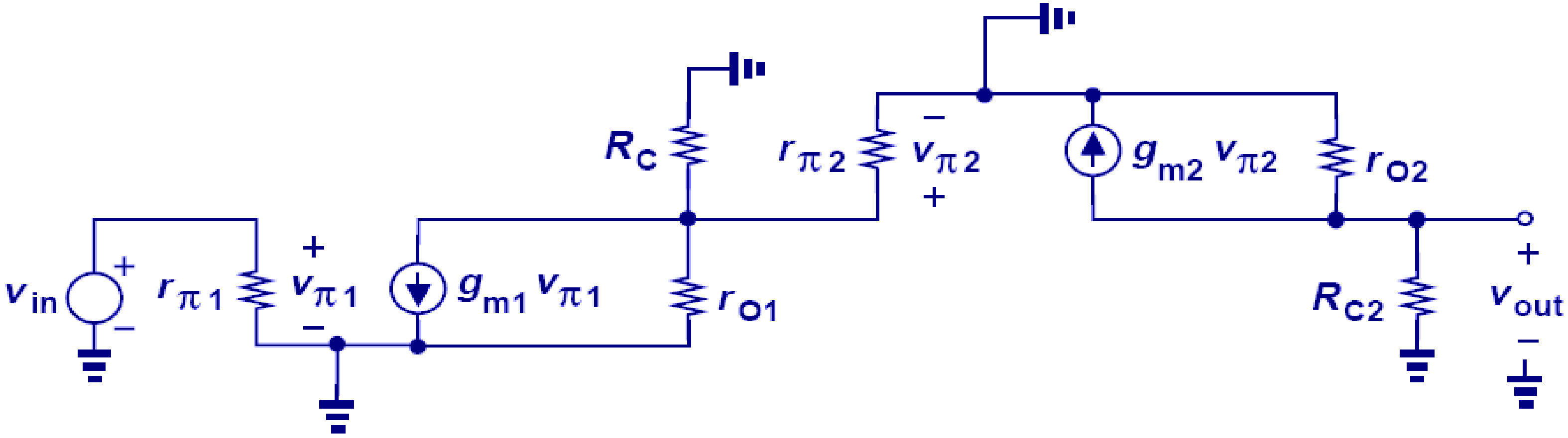
(c)



Small Signal Model Example IV



(a)



(b)

Bonus HW: Derive its A_V (= v_{out}/v_{in}) =?

Analog Signals and Linear Amplifiers

- Analog signal :
  - The magnitude can take on any value within limits and may vary continuously with time.
- Analog circuit :
  - Electronic circuits process analog signal.
- Linear amplifier :
  - produce an output signal whose magnitude is proportional to the input signal.
- Two analysis types of the amplifier
  - DC : ac source set to zero or large signal analysis
  - AC : dc source set to zero or small signal analysis
- Superposition principle:
  - The total response is the sum of the ac and dc individual response.

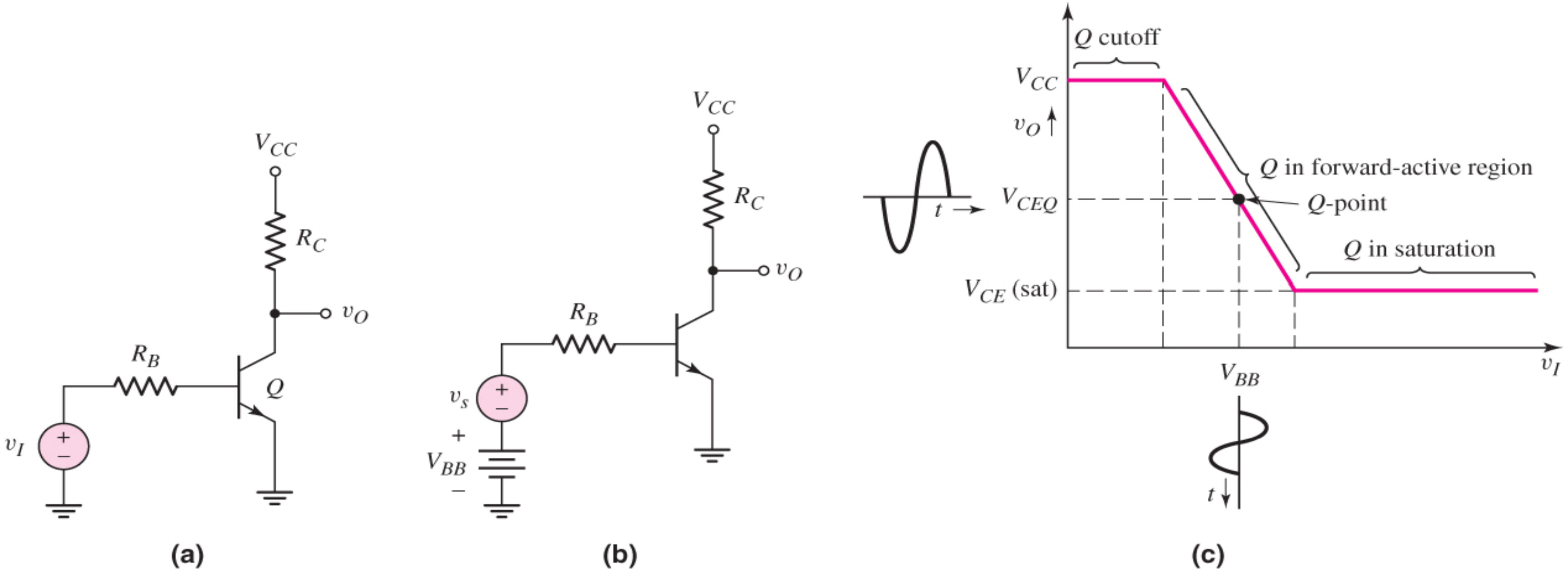
What is a linear system?

The Bipolar Linear Amplifier

- Bipolar inverter amplifier

More amplifiers will be discussed later.

  - Transistor Biased at :
    - Forward-active region for amplification $v_O = A_V \cdot v_s$
    - Cutoff or saturation : V_O is independent to V_S

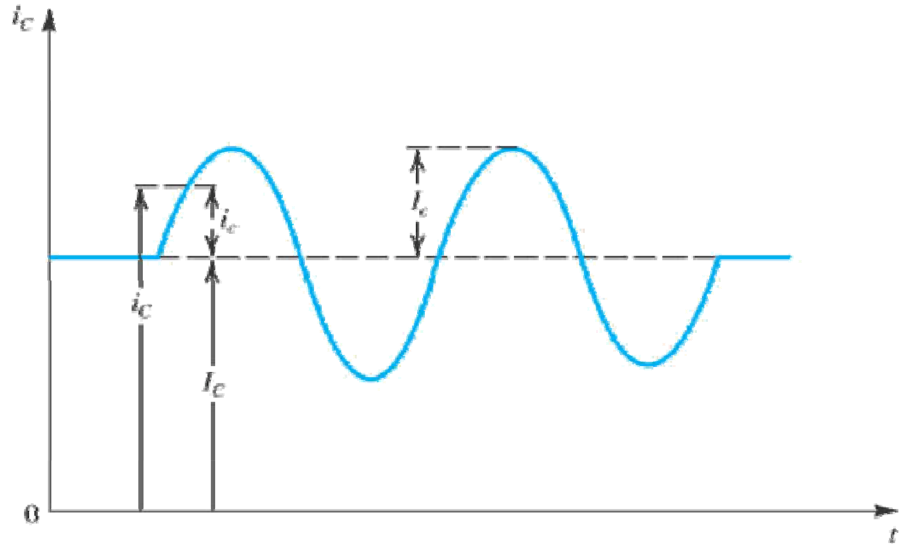


Signal Notation

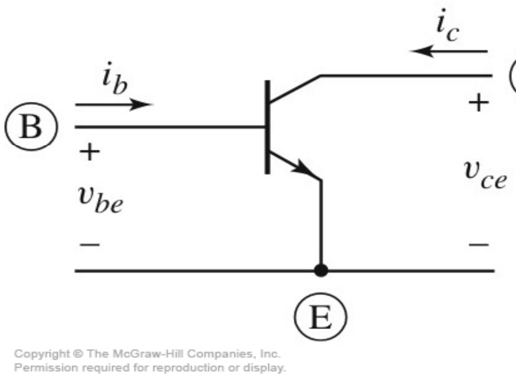
- A lowercase letter with an uppercase subscript, such as i_B or v_{BE}, indicates *total instantaneous values*.
- An uppercase letter with an uppercase subscript, such as I_B or V_{BE}, indicates *dc quantities*.
- A lowercase letter with a lowercase subscript, such as i_b or v_{be}, indicates instantaneous values of *ac signals*.
- An uppercase letter with a lowercase subscript, such as I_b or V_{be}, indicates *phasor quantities*.

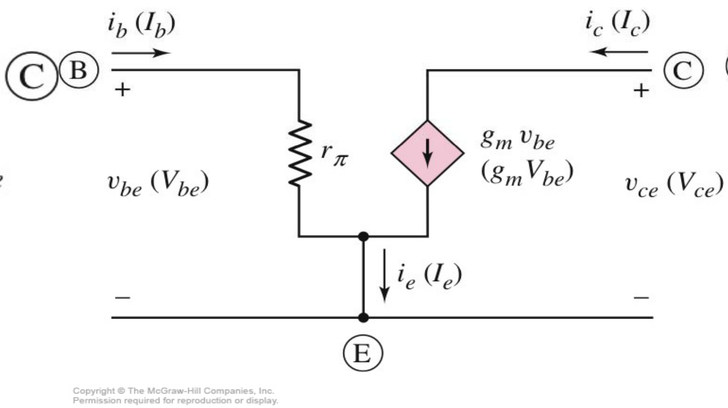
Table 6.1 Summary of notation

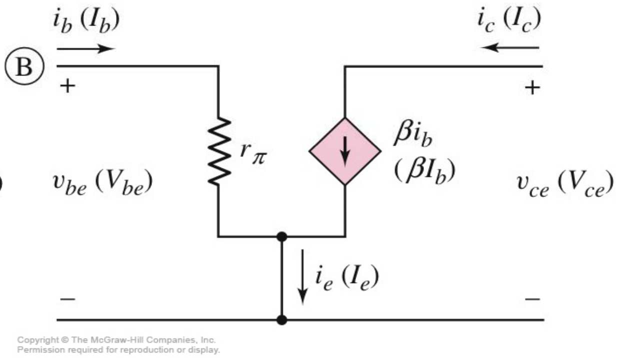
Variable	Meaning
i_B, v_{BE}	Total instantaneous values
I_B, V_{BE}	DC values
i_b, v_{be}	Instantaneous ac values
I_b, V_{be}	Phasor values



Small-signal Hybrid-π Equivalent Circuit of BJT







- Diffusion resistance :

$$r_\pi \equiv \left(\left. \frac{\partial i_B}{\partial \upsilon_{be}} \right|_{Q-pt} \right)^{-1} = \left(\frac{I_{BQ}}{V_T} \right)^{-1}$$

- Transconductance :

$$g_m \equiv \left(\left. \frac{\partial i_C}{\partial \upsilon_{be}} \right|_{Q-pt} \right)^{-1} = \frac{I_{CQ}}{V_T}$$

- C-E current gain :

$$r_\pi \cdot g_m = \left(\frac{I_{BQ}}{V_T} \right)^{-1} \cdot \left(\frac{I_{CQ}}{V_T} \right) = \left(\frac{I_{CQ}}{\beta V_T} \right)^{-1} \cdot \left(\frac{I_{CQ}}{V_T} \right)$$

$$= \beta \text{ (is constant!)}$$

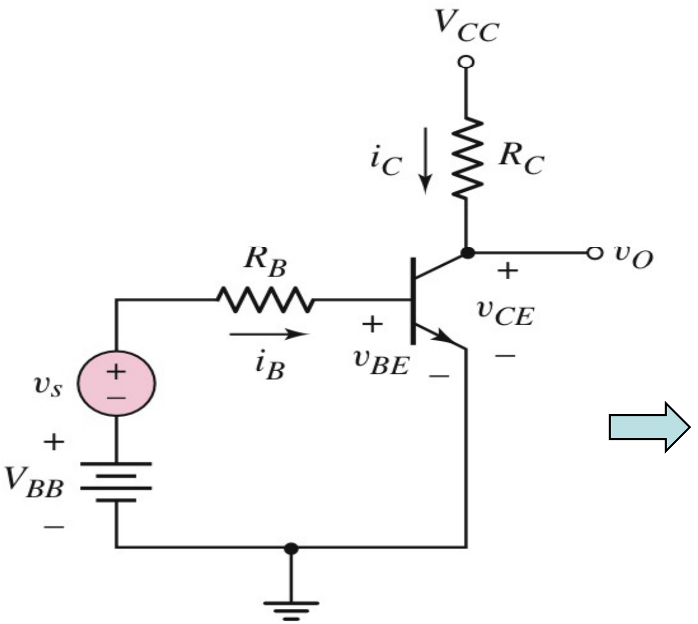
Small-signal Equivalent Circuit

- π-model is shown within the dotted lines
- Small signal voltage gain :

$$A_\upsilon = \frac{V_o}{V_s}$$

$$V_o = -(g_m V_\pi) R_C, (\because V_\pi = \left(\frac{r_\pi}{r_\pi + R_B}\right) \cdot V_s)$$

$$\Rightarrow A_\upsilon = -(g_m R_C) \cdot \left(\frac{r_\pi}{r_\pi + R_B}\right)$$



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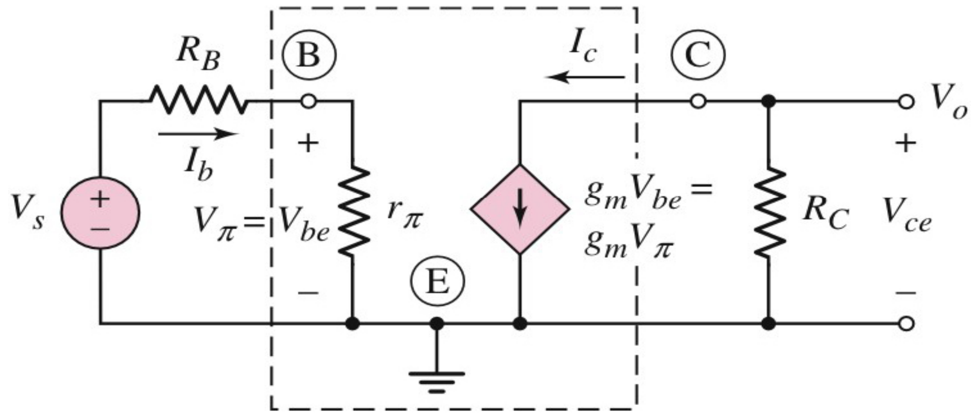

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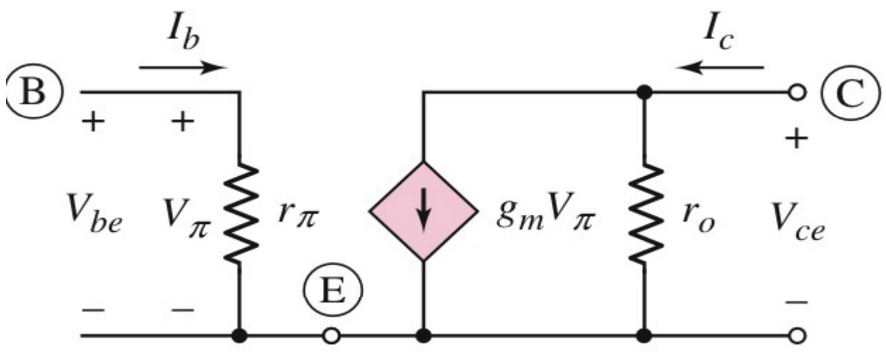
Hybrid-π Equivalent Circuit with Early Effect (V_A)



(a)

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- r_o: small-signal transistor output resistance

$$r_o \equiv \left(\left. \frac{\partial i_C}{\partial \upsilon_{CE}} \right|_{Q-pt} \right)^{-1} \cong \left(\frac{I_{CQ}}{V_A} \right)^{-1}$$

* Expanded Hybrid-π Equivalent Circuit

- Expanded hybrid-π equivalent circuit includes two additional resistances, r_b and r_μ.
- r_b is the **series resistance** of the semiconductor material between the external base terminal B and an idealized internal base region B.
- r_b is a few tens of ohms and is usually much smaller than r_π ;
- r_b is normally negligible (a short circuit) at low frequencies.
- At high frequencies, r_b may not be negligible, since the input impedance becomes capacitive.

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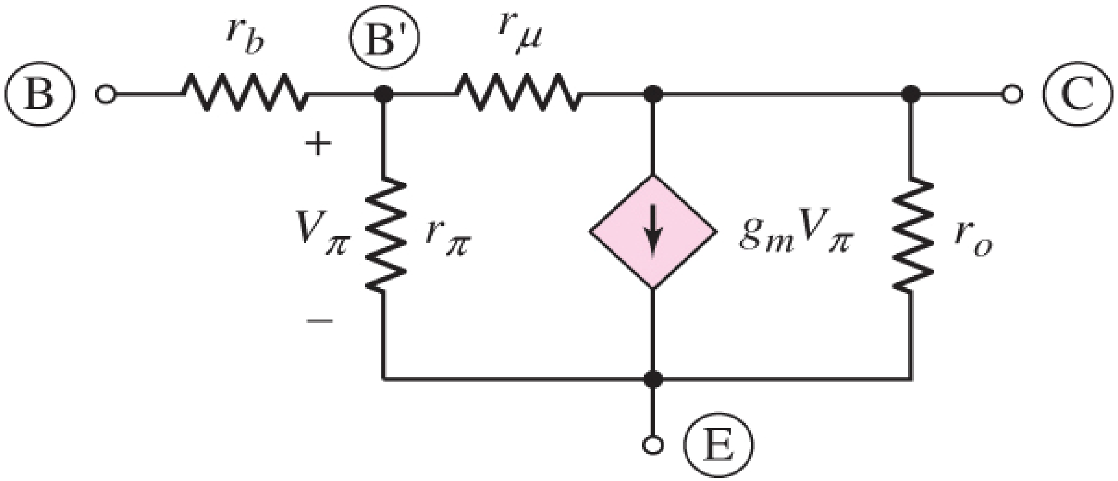

- r_μ is the **reverse-biased diffusion resistance** of the base–collector junction, typically on the order of meg-ohms and can be neglected (an open circuit).
- The resistance does provide some feedback between the output and input, meaning that the base current is a slight function of the collector–emitter voltage.
- In this text, Hybrid-π equivalent circuit model is used, neglect both r_b and r_μ, unless they are specifically included.

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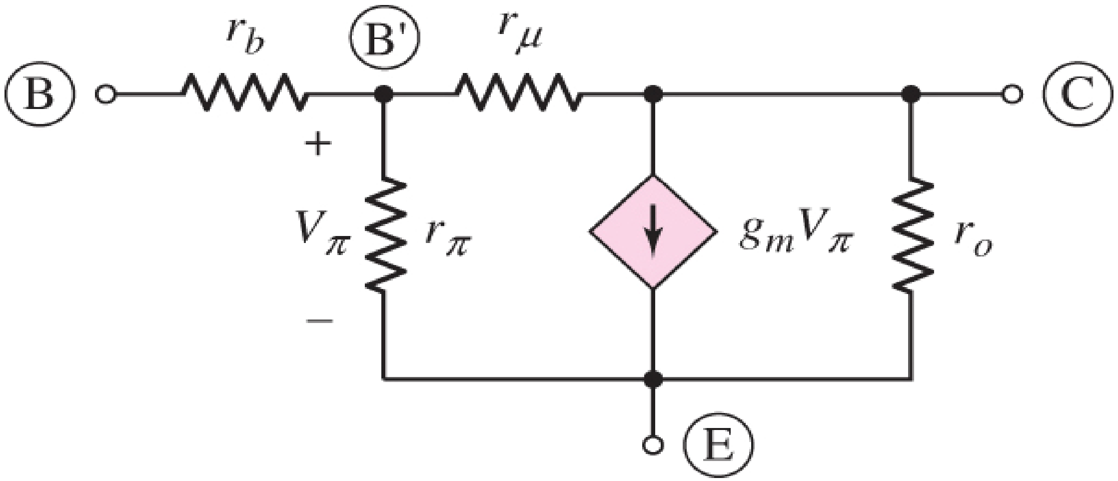

* Other Small-Signal Equivalent Circuits

- *h*-parameters
  - Relate the small-signal terminal currents and voltages of a two-port network
  - Normally given in bipolar transistor data sheets, and are convenient to determine experimentally at low frequency.
  - If we assume the transistor is biased at a *Q*-point in the forward-active region, the linear relationships between the small-signal terminal currents and voltages can be written as
    - $V_{be} = h_{ie} \cdot I_b + h_{re} \cdot V_{ce}$
    - $I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_{ce}$

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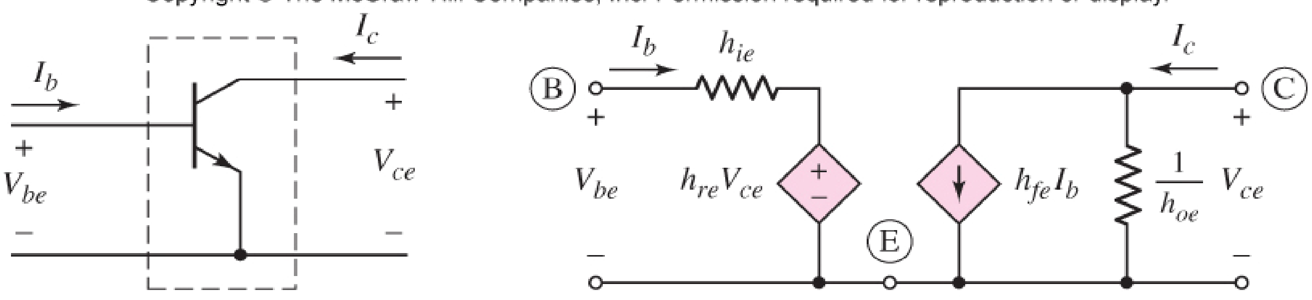

(a) (b)

where the subscripts are: *i* for input, *r* for reverse, *f* for forward, *o* for output, and *e* for common emitter.

- Small-signal input resistance h_{ie} is $h_{ie} = r_b + r_\pi \| r_\mu \cong r_\pi$
- The parameter h_{fe} is the small-signal current gain and is found to be

$$h_{fe} = g_m r_\pi = \beta$$

- The small-signal output admittance h_{oe} is given by $h_{oe} \cong \dfrac{1}{r_o}$
- h_{re} is called the **voltage feedback ratio** and can be written as

$$h_{re} \cong \frac{r_\pi}{r_\pi + r_\mu} \approx 0$$

- The *h*-parameters for a pnp transistor are defined in the same way as those for an npn device.
- small-signal equivalent circuit for a pnp transistor using *h*-parameters is identical to that of an npn device, except that the current directions and voltage polarities are reversed.